PHYS222

Uncertainty calculations for index of refraction calculations

Snell's Law is $n_i \sin\theta_i = n_r \sin\theta_r$ and can be written as $\frac{n_r}{n_i} = \frac{\sin\theta_i}{\sin\theta_r}$

If we look at a ray going from air into our plastic block ($n_i = n_{air} = 1.0003 = 1.000$) Eq-1 becomes

$$n_r = \frac{\sin\theta_i}{\sin\theta_r}. \qquad \text{Eq-1}$$

The fractional uncertainty for Eq-2 is given by *(where uncertainty of is considered negligible)*

$$\frac{\delta n_r}{n_r} = \sqrt{\left(\frac{\delta \sin\theta_i}{\sin\theta_i}\right)^2 + \left(\frac{\delta \sin\theta_r}{\sin\theta_r}\right)^2} \qquad \text{Eq-2}$$

By definition $\delta \sin\theta = \left|\frac{d \sin\theta}{d\theta}\right| \delta\theta = |\cos\theta|\,\delta\theta$.

The fractional uncertainty can thus be written as $\frac{\delta \sin\theta}{|\sin\theta|} = |\cot\theta|\,\delta\theta$ Eq-3

Substituting Eq-3 into Eq-2 we have

$$\boxed{\frac{\delta n_r}{n_r} = \sqrt{\left[(\cot\theta_i)(\delta\theta)\right]^2 + \left[(\cot\theta_r)(\delta\theta)\right]^2}} \qquad \text{Eq-4}$$

This is the equation you use. –(Note above that $\delta\theta_i = \delta\theta_r = \delta\theta = 0.5$degrees and $\delta\theta$ **must be in radians**).

Example

For a light ray which travels from air into a plastic block the following measurements were made:

$\theta_i = 33.0 \pm 0.5$ degrees, $\theta_r = 22.0 \pm 0.5$ degrees, $\delta\theta = 0.5$ degrees $= 0.00873 rads$

The calculated $n_r = 1.454$ (from Snell's Law)

Substituting the values above into Eq-4 we have

$$\frac{\delta n_r}{1.454} = \sqrt{\left[(\cot 33)(0.00873 rads)\right]^2 + \left[(\cot 22)(0.00873 rads)\right]^2}$$

$$\delta n_r = 1.454(0.02545) = 0.0370$$

$$\delta n_r = 0.04$$

Thus $n_r = 1.45 \pm 0.03$ or $n_r = 1.45 \pm 2\%$

Technically, you should do the uncertainty calculations for each set of calculations and combine your results. For brevity you will do only one uncertainty calculation and use that value for the "average uncertainty" of all your measurements. **This uncertainty calculation will be on quiz next week!!!**