Phys 735 – Gravitational Physics
Optional Homework Assignment #7. Due Friday, Dacember 9, 2022

In your solutions to the following problems, please include full sentences explaining your reasoning. Any step that is not an obvious mathematical manipulation should come with a short explanation, or the solution will not be considered complete.

1. Schwarzschild geometry:

(a) Starting from the Schwarzschild metric written in the usual (t, r, θ, ϕ) coordinates, derive the form of the metric if we use (u, r, θ, ϕ) as coordinates instead, where u is the "outgoing" Eddington-Finkelstein coordinate

$$u := t - r - 2GM \ln \left(\frac{r}{2GM} - 1 \right).$$

(b) Show that a diagram in the (u, r) plane showing the Schwarzschild light cones is as shown in Carroll's Figure 5.11, and argue that this diagram represents the white hole portion of the Schwarzschild spacetime.

2. Motion and signals in Schwarzschild spacetime:

A clock is in a circular orbit at $r = 10\, M$ in a Schwarzschild metric.

(a) How much time elapses on the clock during one orbit? (Integrate the proper time $\mathrm{d}t = |\mathrm{d}s^2|^{1/2}$ over one orbit.)

(b) The clock sends out a signal to a distant observer once each orbit. What time interval does the distant observer measure between receiving two successive signals?

(c) A second clock is at rest at $r = 10\, M$ next to the orbit of the first clock, kept in place by rockets. How much time elapses on it between successive passes of the orbiting clock?

3. Perturbations of Minkowski space:

(a) Show that under a gauge transformation $h_{\mu\nu} \mapsto h_{\mu\nu} + 2\, \partial_{(\mu}\xi_{\nu)}$ the variables Φ, w_i, Ψ and s_{ij} transform as written in Eqs. (7.33).

(b) Explain, mostly in words and using just a few key equations, why there are only two physical degrees of freedom for perturbations of Minkowski space in general relativity.

4. Carroll, Chapter 7, Exercise 5.