Phys 735 – Gravitational Physics Homework Assignment #3. Due Thursday, October 6, 2022

In your solutions to the following problems, please include full sentences explaining your reasoning. Any step that is not an obvious mathematical manipulation should come with a short explanation, or the solution will not be considered complete.

1. Carroll, Chapter 2, Exercise 7.

- 2. Carroll, Chapter 2, Exercise 8.
- 3. Carroll, Chapter 2, Exercise 9.

4. Distances and volumes:

In a certain spacetime geometry the metric is

$$ds^{2} = -(1 - Ar^{2})^{2} dt^{2} + (1 - Ar^{2})^{2} dr^{2} + r^{2} (d\theta^{2} + \sin^{2}\theta d\phi^{2}) .$$

(a) Calculate the proper distance along a radial line from the center r = 0 to a coordinate radius r = R.

(b) Calculate the surface area of a sphere of coordinate radius r = R, at fixed t.

(c) Calculate the three-volume of a sphere of coordinate radius r = R, at fixed t.

(d) Calculate the four-volume of a four-dimensional tube bounded by a sphere of coordinate radius R and two t = constant planes separated by a coordinate time T.