Phys 727: Thermodynamics and Statistical Mechanics
Spring 2026 – Equations to Remember for Final Exam

The final exam will consist of conceptual questions and simple exercises on the subjects we have covered since Test 2 in class. Students will be expected to be able to define all concepts introduced in class and explain their significance in words (in addition to the main ones introduced earlier in the course that are used here—see the Test 2 study guide for those), and write down proofs that can be reproduced in a couple of steps. The equations and relationships to be remembered are listed below.

The numbers on the left side of the list correspond to the lecture notes where the topics are covered.

Mixed Quantum States and Particle Statistics

13 Mixed Quantum States

- Density matrices — $\rho : \mathcal{H} \to \mathcal{H}$ satisfying $\rho^\dagger = \rho$ and $\mathrm{tr}\,\rho = 1$; those which also satisfy $\rho^2 = \rho$ are equivalent to Hilbert space vectors $|\psi\rangle$ such that $\rho = |\psi\rangle\langle\psi|$
- Components in a basis $\{\phi_\alpha\}$ — $\rho_{\alpha\alpha'} = \langle\phi_{\alpha'} \mid \rho \mid \phi_\alpha\rangle$
- Observables — $\hat{A} : \mathcal{H} \to \mathcal{H}$, with mean value $\langle\hat{A}\rangle = \mathrm{tr}\,\rho\hat{A}$ in a state ρ
- Time evolution — Liouville-von Neumann equation, $\partial_t\,\rho = (\mathrm{i}\hbar)^{-1}[\hat{H}, \rho]$
- Quantum partition function — $Z_\mathrm{c} = \mathrm{tr}\,\mathrm{e}^{-\beta\hat{H}}$ or $Z_\mathrm{g} = \mathrm{tr}\,\mathrm{e}^{-\beta(\hat{H}-\mu\hat{N})}$

14 Many-Particle States

- Boson and Fermion wave functions — $\psi(\mathbf{r}_{P1}, ..., \mathbf{r}_{PN}) = (\pm 1)^{[P]}\,\psi(\mathbf{r}_1, ..., \mathbf{r}_N)$
- Single-particle plane-wave basis — $\phi_\mathbf{k}(\mathbf{r}) = V^{-1/2}\,\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}$, where $\mathbf{p} = \hbar\mathbf{k}$; with periodic boundary conditions $k_i = (2\pi/L_i)\,n_i$, where each $n_i \in \mathbf{Z}$
- Basis — $\phi_{\{\alpha\}}(\mathbf{r}_1, ..., \mathbf{r}_N) = (N!)^{-1/2} \sum_{P\in\mathcal{P}_N} (\pm 1)^{[P]}\,\phi_{\alpha_1}(\mathbf{r}_{P1}) \cdots \phi_{\alpha_N}(\mathbf{r}_{PN})$ if all state labels α_is are different
- Fock-space representation — $|N_{\alpha_1}, ..., N_{\alpha_i}, ...\rangle$, $\sum_\alpha N_\alpha = N$, $\sum_\alpha N_\alpha \epsilon_\alpha = E$
- Quantum statistics — mean occupation numbers $\bar{N}(\epsilon) = [\mathrm{e}^{(\epsilon-\mu)\beta} \mp 1]^{-1}$
- Density of states — used for $\bar{N} \approx \int \mathrm{d}\epsilon\, g(\epsilon)\,\bar{N}(\epsilon)$, $\bar{E} \approx \int \mathrm{d}\epsilon\, g(\epsilon)\,\epsilon\,\bar{N}(\epsilon)$; be familiar with the steps needed to calculate $g(\epsilon)$

Ideal Quantum Systems

15 Monatomic Ideal Gas

- Partition function — When it can be approximated by an integral
- Semiclassical approximation — Be able to reproduce steps in the derivation of the partition function as a sum of the classical one plus correction terms (but not the whole derivation), and when the approximation is a good one (definition and interpretation of the degeneracy parameter δ)

16 Diatomic molecules

- Why we can write $Z = Z_\mathrm{t}\,Z_\mathrm{r}\,Z_\mathrm{v}$ and $C_V = C_\mathrm{t} + C_\mathrm{r} + C_\mathrm{v}$; how to calculate each partition function and and how the quantum mechanical results for C_r and C_v differ from the classical ones

17 Massive particles

- Density of states — Be able to calculate the form of $g(\epsilon)$
- The free boson gas — Be able to reproduce individual steps in the calculation of $\bar{N}$ and $\bar{E}$ (but not the special integrals or functions involved) and explain what Bose-Einstein condensation is and how it arises in the calculation

18 Massless particles

- Density of states — Be able to calculate the form of $g(\omega)$
- The photon gas — Be able to reproduce individual steps in the calculation of Z, F, S, $\bar{E}$, C_V, p (but not the special integrals or functions involved)
- Ultrarelativistic particles — same