

Phys 727: Thermodynamics and Statistical Mechanics
Spring 2026 – Equations to Remember for Test 2

The test will consist of conceptual questions or simple exercises on all subjects we have covered since Test 1 in class. Students will be expected to remember the definitions and other equations listed below (some parts of the Test 1 material are included), and to be familiar with all concepts and explanations introduced in class, be able to define them and explain their significance (not all of these are listed below).

The numbers on the left side of the list correspond to the lecture notes where the topics are covered.

Thermodynamics

- 01 Fundamental relations $\Delta E = Q + W$ and statements of the three laws; $T := (\partial S / \partial E |_{\vec{X}})^{-1}$
 fundamental identity $dE = T dS + \vec{f} \cdot d\vec{X}$ [e.g., $-p dV$, $-\vec{M} \cdot d\vec{B}$, ...]; when $\delta Q = T dS$
- 02 Potentials $F := E - TS$ $H := E + pV$ $E = TS - pV + \sum_i \mu_i N_i$
 $G := E - TS + pV$ $G = \sum_i \mu_i N_i$ $\Omega := E - TS - \mu N$
 plus the fundamental identity and how to calculate first-order quantities in each case
- 03 Response functions Be able to derive Maxwell relations from the thermodynamic identity
 $C_V = \delta Q / \delta T |_V = T (\partial S / \partial T)_V$ $C_p = \delta Q / \delta T |_p = T (\partial S / \partial T)_p$
 $\kappa_T = -V^{-1} (\partial V / \partial p)_T$ $\kappa_S = -V^{-1} (\partial V / \partial p)_S$ $\gamma = C_p / C_V$ (also κ_T / κ_S)
- 04 Ideal gas $pV = Nk_B T$ $E = (f/2) Nk_B T$
 Non-ideal gas Virial expansion: $p = k_B T \rho [1 + B_2(T)\rho + \dots]$
 Van der Waals fluid What it is, physically; phase transition; no need to remember the equations of state

Statistical Mechanics

- 07 Phase space Γ volume $\omega(R) = \int_R d\omega$ and number of states $\Omega(R) = \int_R d\Omega$ for $R \subset \Gamma$,
 with $d\omega = d^{3N}q d^{3N}p$ and $d\Omega = (h^{3N} N!)^{-1} d^{3N}q d^{3N}p$ for N particles in 3D
- 08 Microcanonical state $\rho(q, p) = \text{constant}$, from the equal probabilities assumption
 Boltzmann entropy $S_B(E, \vec{X}) = k_B \ln \Omega_{\vec{X}}(E, \Delta)$, and $T^{-1} = \partial S / \partial E$
- 09 Canonical state $\rho(s) = \frac{1}{Z} e^{-\beta H(s)}$, $Z = \sum_s e^{-\beta H(s)}$ or $\int_{\Gamma} d\Omega e^{-\beta H(p, q)}$,
 Gibbs entropy $S_G = -k_B \sum_s \rho(s) \ln \rho(s)$,
 with thermodynamics obtained from $\bar{E} = -\frac{\partial}{\partial \beta} \ln Z$ and $F = -k_B T \ln Z$
- 10 Monatomic ideal gas $Z_1 = V / \lambda_T^3$, $Z_N = V^N / \lambda_T^{3N} N!$, $\lambda = \sqrt{h^2 \beta / 2\pi m}$; Equipartition principle
- 11 Simple systems Be able to write down the Hamiltonian and derive the main thermodynamic
 quantities for paramagnets and simple harmonic oscillators;
 Difference between Boltzmann, Bose-Einstein and Fermi-Dirac statistics;
 Partition function and thermodynamics for a system with uncoupled subsystems
- 11 Grand canonical state $\rho(s) = \frac{1}{Z_g} e^{-\beta(H - \mu N)}$, $Z_g = \sum_{N, s} e^{-\beta(H - \mu N)} = \sum_{N=0}^{\infty} z^N Z_N$, $z = e^{\mu \beta}$, with
 thermodynamics from $\bar{N} = \frac{1}{\beta} \frac{\partial}{\partial \mu} \ln Z_g$, $\bar{E} = -\frac{\partial}{\partial \beta} \ln Z_g + \mu \bar{N}$, $\Omega = -k_B T \ln Z_g$

Other Physics

Common definitions, expressions, equations from other areas of physics (Poisson brackets; the Hamiltonian formalism for dynamics and form of H for free particles, harmonic oscillators, charged particles, magnetic dipoles, ...).