Phys 727: Thermodynamics and Statistical Mechanics
Spring 2025 – Equations to Remember for Test 2

The test will consist of conceptual questions or simple exercises on all subjects we have covered in class so far, and short problems on thermodymanics topics. Students will be expected to remember the definitions and other equations listed below (some parts of the Test 1 material are included), and to be familiar with all concepts and explanations introduced in class, be able to define them and explain their significance (not all of these are listed below).

The numbers on the left side of the list correspond to the lecture notes where the topics are covered.

Thermodynamics

01 Fundamental relations — $\Delta E = Q + W$ and statements of the three laws; $T := (\partial S/\partial E|_{\vec{X}})^{-1}$
fundamental identity $\mathrm{d}E = T\,\mathrm{d}S + \vec{f}\cdot\mathrm{d}\vec{X}$ [e.g., $-p\,\mathrm{d}V$, $-\vec{M}\cdot\mathrm{d}\vec{B}$, ...]; when $\delta Q = T\,\mathrm{d}S$

02 Potentials — $F := E - TS$ $\quad$ $H := E + pV$ $\quad$ $E = TS - pV + \sum_i \mu_i N_i$
$G := E - TS + pV$ $\quad$ $G = \sum_i \mu_i N_i$ $\quad$ $\Omega := E - TS - \mu N$
plus the fundamental identity and how to calculate first-order quantities in each case

03 Response functions — Be able to derive Maxwell relations from the thermodynamic identity
$C_V = \delta Q/\delta T|_V = T\,(\partial S/\partial T)_V$ $\quad$ $C_p = \delta Q/\delta T|_p = T\,(\partial S/\partial T)_p$
$\kappa_T = -V^{-1}\,(\partial V/\partial p)_T$ $\quad$ $\kappa_S = -V^{-1}\,(\partial V/\partial p)_S$ $\quad$ $\gamma = C_p/C_V$ (also κ_T/κ_S)

04 Ideal gas — $pV = Nk_\mathrm{B}T$ $\quad$ $E = (f/2)\,Nk_\mathrm{B}T$

Non-ideal gas — Virial expansion: $\quad p = k_\mathrm{B}T\rho\left[1 + B_2(T)\rho + \dots\right]$

Van der Waals fluid — What it is, physically; phase transition; no need to remember the equations of state

Statistical Mechanics

07 Phase space — volume $\omega(R) = \int_R \mathrm{d}\omega$ and number of states $\Omega(R) = \int_R \mathrm{d}\Omega$ for $R \subset \Gamma$,
with $\mathrm{d}\omega = \mathrm{d}^{3N}q\,\mathrm{d}^{3N}p$ and $\mathrm{d}\Omega = (h^{3N}N!)^{-1}\mathrm{d}^{3N}q\,\mathrm{d}^{3N}p$ for N

08 Microcanonical state — $\rho(q,p) =$ constant, from the equal probabilities assumption
Boltzmann entropy $S(E,\vec{X}) = k_\mathrm{B}\ln\Omega_{\vec{X}}(E,\Delta)$, and $T^{-1} = \partial S/\partial E$

09 Canonical state — $\rho(s) = \dfrac{1}{Z}\,\mathrm{e}^{-\beta H(s)}\,, \quad Z = \sum_s \mathrm{e}^{-\beta H(s)}$ or $\displaystyle\int_\Gamma \mathrm{d}\Omega\,\mathrm{e}^{-\beta H(p,q)}$, $\quad$ Gibbs entropy
with thermodynamics obtained from $\bar{E} = -\frac{\partial}{\partial\beta}\ln Z$ and $F = -k_\mathrm{B}T\ln Z$

10 Monatomic ideal gas — $Z_1 = V/\lambda_T^3\,, \quad Z_N = V^N/\lambda_T^{3N}N!\,, \quad \lambda = \sqrt{h^2\beta/2\pi m}\,;$ $\quad$ Equipartition principle

11 Simple systems — Be able to write down the Hamiltonian and derive the main thermodynamic quantities for paramagnets and simple harmonic oscillators

11 Grand canonical state — $\rho(s) = \dfrac{1}{Z_\mathrm{g}}\,\mathrm{e}^{-\beta(H-\mu N)}\,, \qquad Z_\mathrm{g} = \sum_{N,s}\mathrm{e}^{-\beta(H-\mu N)} = \sum_{N=0}^{\infty}\mathrm{e}^{\beta\mu N}Z_N$

$$\bar{N} = \frac{1}{\beta}\frac{\partial}{\partial\mu}\ln Z_\mathrm{g}\,, \quad \bar{E} = -\frac{\partial}{\partial\beta}\ln Z_\mathrm{g} + \mu\bar{N}\,, \quad \Omega = -k_\mathrm{B}T\ln Z_\mathrm{g}$$

16 Molecular gas — Why we can write $Z = Z_\mathrm{t}\,Z_\mathrm{r}\,Z_\mathrm{v}$ and $C_V = C_\mathrm{t} + C_\mathrm{r} + C_\mathrm{v}$; how to calculate each;
How the quantum mechanical results for C_r and C_v differ from the classical ones

Other Physics

Common definitions, expressions, equations from other areas of physics (Poisson brackets; the Hamiltonian formalism for dynamics and form of H for a free particle, a harmonic oscillator, a magnetic dipole in a magnetic field, ...).