Physics 652: Assignment 1

(to be submitted by Tuesday, February 7, 2023)

1. Use this *Mathematica* command

```
DSolve[y'[x] == a y[x] (1 - y[x]/Y), y[x], x]
```

   to obtain the solution to the *logistic equation*,

$$\dot{y} = \frac{dy}{dt} = ay\left(1 - \frac{y}{Y}\right).$$

   Try out this next code snippet to check that the purported solution actually solves the logistic equation:

```
rhslogisticeqn = a y[x] (1 - y[x]/Y)
soln = First[DSolve[y'[x]==rhslogisticeqn, y[x], x]]
Simplify[D[y[x]/.soln,x]==rhslogisticeqn/.soln]
```

   Now show explicitly (by hand) that

$$y(t) = \frac{e^{ax+bY}Y}{e^{ax+bY} - 1} = \frac{Y y_0 e^{at}}{Y + y_0(e^{at} - 1)}$$

   is a solution to the ODE.

2. Separate the variables of $(1 + y^2)y\,dx + (1 + x^2)x\,dy = 0$. Find its general integral and solution $y(x)$.

3. Determine whether

$$(1 + x^2 + y^2)^{-3/2}\left[(1 + y^2)y\,dx + (1 + x^2)x\,dy\right] = 0$$

   is exact. Find its general integral and solution $y(x)$. Explain the connection to this *Mathematica* command:

```
DSolve[y'[x] == -(1 + y[x]^2) y[x]/((1 + x^2) x), y[x], x]
```

4. Obtain the general solution to the differential equation $y' + y/x = c/x$ with machine assistance. Then try to arrive at the solution by hand.

```
DSolve[y'[x] + y[x]/x == c/x, y[x], x]
```

5. Obtain the general solution to the differential equation $y' + xy = ce^{-x^2/2}$ with machine assistance. Then try to arrive at the solution by hand.

```
DSolve[y'[x] + x y[x] == c Exp[-x^2/2], y[x], x]
```