Electron – Phonon interaction

* The physical mechanism for superconductivity took nearly 50 years to sort out.

   → huge theoretical challenge to explain the tiny energy scales involved $\sim 10^{-5}$ eV per electron compared to a Coulomb scale ~ 1 eV per electron

   → first hint as to the origin of superconductivity was the discovery of the isotope effect: i.e. different isotopes of the same metal have different critical temperatures T_c below which they show zero resistance and T_c obeys the relation

$$T_c M^{\alpha} = \text{const with } \alpha \approx 0.5$$

Assuming a value[2] of 0.32080 for the ratio $\nu(B^{11})/\nu(H^1)$, one obtains

$$\nu(Nb^{93})/\nu(H^1) = 0.24441 \pm 0.00013.$$

This value should be increased by a Lamb[3] correction of 0.38 percent because of the diamagnetic effects of the atomic electrons. Use of this correction and of the value 5.58504 for the g-factor of the proton[4] leads to the following value for the nuclear g-factor of Nb^{93}:

$$g(Nb^{93}) = 1.3702 \pm 0.0007.$$

Since the spin of Nb^{93} is known from hyperfine structure data[5] to be 9/2, the nuclear magnetic moment of Nb^{93} is

$$\mu(Nb^{93}) = 6.1659 \pm 0.0032 \text{ nuclear magnetons.}$$

These values are to be compared with the earlier values $g=1.18$ and $\mu=5.31$ obtained by Meeks and Fisher[6] from hyperfine structure studies. The value obtained in the present work lies close to the upper Schmidt limit value of $+6.79$ for a $g_{9/2}$ odd proton, which Nb^{93} has according to Mayer's[7] theory.

We wish to express our appreciation to the Ohio State University Development Fund and Research Foundation for grants that made this work possible.

* AEC Research Fellow.
[1] J. R. Zimmerman and D. Williams, Phys. Rev. 76, 350 (1949).
[2] W. H. Chambers and D. Williams, Phys. Rev. 76, 638 (1949).
[3] W. E. Lamb, Phys. Rev. 60, 817 (1941).
[4] J. E. Mack, Rev. Mod. Phys. 22, 64 (1950).
[5] S. S. Ballard, Phys. Rev. 46, 806 (1934).
[6] W. W. Meeks and R. A. Fisher, Phys. Rev. 72, 451 (1947).
[7] M. Mayer, Phys. Rev. 75, 1969 (1949).

Isotope Effect in the Superconductivity of Mercury*

EMANUEL MAXWELL
National Bureau of Standards, Washington, D. C.
March 24, 1950

THE existence of a small quantity of Hg^{198} at the National Bureau of Standards[1] prompted us to investigate its properties as a superconductor.[2] The sample available to us had a high degree of isotopic separation and was approximately 98 percent pure Hg^{198}. The average atomic weight[3] of natural mercury is 200.6. The mercury had been produced by the transmutation of gold and had been prepared by distilling it off the bombarded gold foil.

Preliminary results are now available on the critical field behavior and transition temperature. The destruction of superconductivity was detected magnetically by a ballistic galvanometer method and the zero-field transition temperature determined by extrapolation of the critical field curve to zero field. (Further details of the experimental method will appear elsewhere.) Tests were made with natural mercury and with two specimens of Hg^{198} about 400 mg each (both derived from the original sample). The natural mercury was prepared by our Chemistry Division and is presumed to contain less than 0.001 percent impurity. The specimens of Hg^{198} were separately redistilled in vacuum (following the original distillation from the gold foil) and were enclosed in Pyrex capillaries which were evacuated and sealed off. Temperatures were measured with a helium vapor pressure thermometer using the tables prepared at the Royal Society Mond Laboratory (June 4, 1949) for reducing the pressures to degrees Kelvin.

The results are indicated in Fig. 1 which is a plot of current in the Helmholtz coils (at critical field) vs. the absolute temperature for both the natural mercury and Hg^{198}. Any small difference in slope is not significant since the demagnetization factors of the samples and the exact disposition in the field may have been different. The intercepts are significant and give the transition temperatures as 4.156°K for natural mercury and 4.177°K for Hg^{198}.
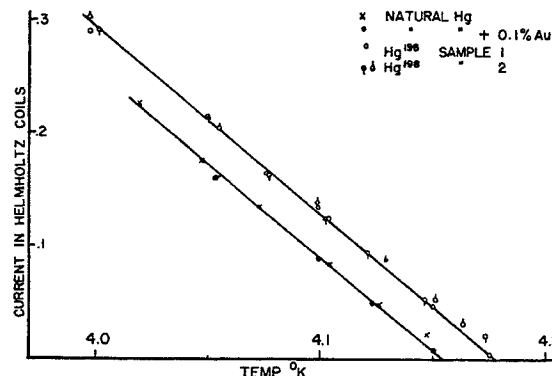

FIG. 1. Current in the Helmholtz coils at the critical field vs. the absolute temperature.

Many pains were taken to exclude the possibility of secondary effects. The earth's field was compensated to within 10^{-4} gauss. The distillation apparatus and capillaries were carefully cleaned. The clean appearance of the mercury was good evidence of the absence of base metal impurities in excess of 0.001 percent but there was some possibility that the Hg^{198} might have contained a gold impurity, derived from the original foil which would not have shown up in the form of surface sum. Spectrochemical analysis of the Hg^{198} was not feasible, so a sample of natural mercury with 0.1 percent gold was prepared and tested. (A saturated solution of gold in mercury will contain about 0.15 percent.) The results are given by the black dots and show that even this large amount of gold impurity is unimportant.

From these results one may infer that the transition temperature of a superconductor is a function of the nuclear mass, the lighter the mass the higher the transition temperature. At the ONR conference in Atlanta, March 20–21, it was reported that Serin and Reynolds[4] at Rutgers University had undertaken a similar investigation independently and simultaneously with our own. They found that the transition temperatures of Hg^{199}, Hg^{202}, and Hg^{204} were also a function of the atomic mass, with the lighter isotopes having higher transition temperatures. It is gratifying to find that our results are mutually consistent and complementary. Taken together they definitely establish a dependence of transition temperature on mass number.

* Supported by the ONR Contract NA-onr 12-48.
[1] We are indebted to Dr. W. F. Meggers for making this sample available to us.
[2] The suggestion of examining a pure isotope was also made to us independently by Professor K. F. Herzfeld.
[3] A. O. Nier, Phys. Rev. 52, 933 (1937).
[4] Reynolds, Serin, Wright, and Nesbitt, Phys. Rev. 78, 487 (1950).

Magic Numbers and Elements with No Stable Isotopes

LEW KOWARSKI
Laboratoire du Fort de Châtillon, Paris, France
March 20, 1950

THAT the abnormal instability of all isotopes of $_{43}$Tc and $_{61}$Pm may have something to do with the occurrence, close by, of magic neutron numbers 50 and 82, is an obvious enough suggestion. It becomes even more plausible if we consider that only the absence of odd-mass isotopes requires a special explanation (even-mass isotopes are, for $Z=43$ and 61, excluded by parity rules). Two other elements, A and Ce, then appear to share the same anomaly and in both the latter cases the connection with the magic neutron numbers (20 and 82, respectively) is even more evident.[1,2]

the observation that 70 percent of the closely spaced double pulses are followed by a third pulse interpretable as a μ-decay while only 30 percent of all the sweeps show a second pulse.

(d) By extrapolation of the decay curve of Fig. 2 back to zero time, we conclude that about 1.5 π^+-mesons were stopped in each gram of crystal per hour. A Kodak NTB nuclear emulsion placed near the crystal indicated that about 1.2 π^+-mesons were stopped in each gram of emulsion per hour.

The circled points in Fig. 2 show the number of delayed pulses corrected for accidentals. By fitting an exponential curve to these points[1] one finds for the mean life of π^+-mesons the value $\tau = (1.65 \pm 0.33) \times 10^{-8}$ sec. Within the rather large statistical errors involved, this value of the mean life agrees with the value $\left(1.97^{+0.14}_{-0.17}\right) \times 10^{-8}$ sec. obtained by Martinelli and Panofsky[2] on the decay of π^+-mesons in flight, and with the value $\left(1.11^{+0.31}_{-0.22}\right) \times 10^{-8}$ sec. obtained by Richardson[3] on the decay of π^--mesons in flight.

We wish to thank the members of the M.I.T. synchrotron staff who have cooperated in making these measurements. Professors Bruno Rossi and Matthew Sands have contributed much encouragement and sound advice. We thank Mr. W. B. Smith who constructed and assisted in the testing of most of the equipment, and Mr. A. Grubman who grew the stilbene crystals.

1 R. Peierls, Proc. Roy. Soc. A149, 467 (1935).
2 E. A. Martinelli and W. K. H. Panofsky, Phys. Rev. 77, 465 (1950).
3 J. R. Richardson, Phys. Rev. 74, 1720 (1948).

Superconductivity of Isotopes of Mercury*

C. A. REYNOLDS, B. SERIN, W. H. WRIGHT, AND L. B. NESBITT
Rutgers University, New Brunswick, New Jersey
March 24, 1950

THE superconducting transition temperatures of natural lead and the lead isotope obtained from the decay of uranium have been determined from resistance measurements by Kamerlingh Onnes and Tuyn[1] and by Justi.[2] In both instances, no detectable change in transition temperature between the natural element and the isotope was found.

In the measurements to be described, critical field vs. temperature curves for mercury samples enriched in various isotopes were determined by susceptibility measurements. The samples were obtained from the U. S. Atomic Energy Commission, and were prepared by electromagnetic separation. The average mass numbers for the enriched samples (samples 1, 2, 4) are shown in Table I; sample 3 is natural mercury.

The samples varying in mass from 50 to 100 mg, were sealed under helium in capillaries about 0.5 mm i.d., giving samples about 3 cm long. A coil of No. 41 copper wire was wound around each capillary. The samples were placed in a Dewar vessel of liquid helium, and formed the secondary of a mutual inductance with a solenoid placed outside the shield Dewar. The solenoid was excited by a 1000 c/sec. oscillator, and produced a magnetic field at the sample of less than 0.1 oersted. The signal picked up by the sample coil was amplified and detected. The signal was balanced out with the sample superconducting, and then the detected signal

TABLE I. Transition temperatures.

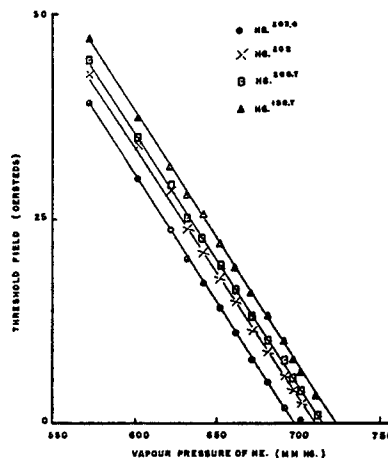| Sample	Average mass number	$T_0(°K)$
1	203.4	4.126
2	202.0	4.143
3	200.7	4.150
4	199.7	4.161

FIG. 1. The critical magnetic field as a function of the absolute temperature.

was observed as a function of the magnetic field of a large Helmholtz coil surrounding the Dewars. The earth's magnetic field was canceled to 0.3 percent. When the magnetic field reached the neighborhood of the critical value at any temperature, there was a rapid increase in detected signal. The critical field was taken to be the average value in the transition interval. Figure 1 shows the critical field curves for the four samples. It is to be noted that there is a *systematic decrease of transition temperature with increasing mass*. The transition temperatures (in zero magnetic field), T_0, of the samples are given in Table I. The slopes of these curves at zero field agree within 1.0 percent; the magnitude of the slope is -204 ± 2 oersteds/°K.

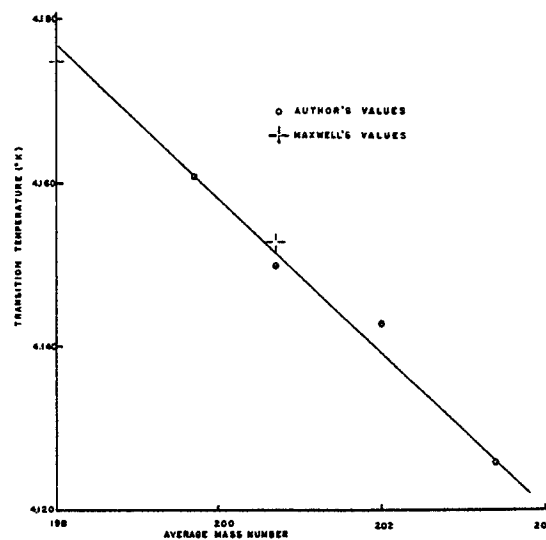Figure 2 is a plot of transition temperature vs. average mass

FIG. 2. The transition temperature vs. the average mass numbers of the isotopic mixtures.

number. The values obtained by Maxwell[3] for Hg[198] and natural Hg are included. The slope of the line in Fig. 2 is 0.009 °K/(mass number).

1 H. Kamerlingh Onnes and W. Tuyn, Leiden Comm. No. 160b (1922).
2 E. Justi, Physik. Zeits. 42, 325 (1941).
3 E. Maxwell, Phys. Rev. 78, 477 (1950).

→ that result suggest that the lattice is an active participant in creating the super-conducting state

* How can electrons interact with each other via phonons (quantized vibrational excitations of the lattice)?

→ phonons are characterized by crystal momentum q and energy $\hbar\omega_q$

→ an electron with wave vector $\vec{k}_1$ can interact with another electron with wave vector $\vec{k}_2$ by exchanging a phonon
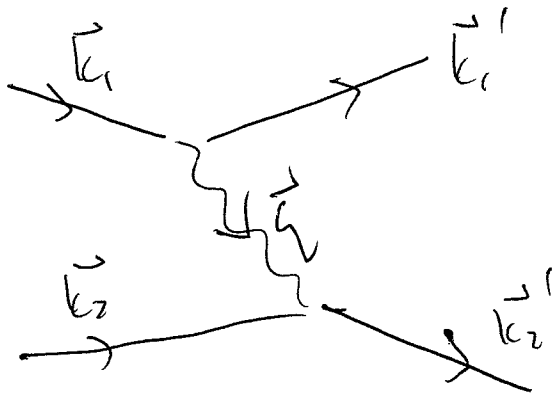
→ by conservation of momentum

$$\vec{k}_1 + \vec{k}_2 = \vec{k}_1' + \vec{k}_2'$$

and

$$\vec{k}_1 = \vec{k}_1' + \vec{q}$$
$$\vec{k}_2' = \vec{k}_2 + \vec{q}$$

→ What is the sign of this effective
electron-electron interaction?

Here's a rough argument:

① When electron $\vec{k}_i$ interacts with the
lattice and is scattered into state $\vec{k}_i'$,
it causes local oscillations of the
electron density at frequency $\omega = (\varepsilon_{k_i} - \varepsilon_{k_i'})/\hbar$

② If at some instance the local electron
density is enhanced at some position
in the crystal (more negative change),
the surrounding ions will start moving
toward the negative excess charge in
order to compensate it

③ But the ion mass is large and the
motion is slow, so there is a tendency
to overcompensate and create net
positive change

④ The second electron with wave vector $\vec{k_2}$ can interact with the positive charge excess; and the interaction can be attractive if the lattice vibrations are in phase with the "driving force" at frequency $\omega = (\mathcal{E}_{\vec{k_1}} - \mathcal{E}_{\vec{k_1'}})/\hbar$ and if $\omega < \omega_D$, the Debye frequency which is the maximum frequency of oscillation for the ions

* Consider a model of a simple harmonic oscillator of mass m, natural frequency ω_0, subject to a driving force $fe^{i\omega t}$.

→ Then the equation of motion is
$$\ddot{x} + \omega_0^2 x = \frac{f}{m} e^{i\omega t}$$

→ A solution of the form $x(t) = x_0 e^{i\omega t}$ gives
$$x_0 (i\omega)^2 + \omega_0^2 x_0 = \frac{f}{m}$$
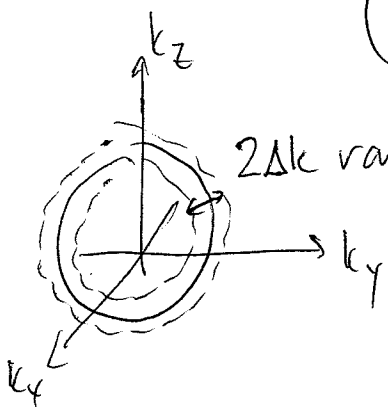
or

$$x_0 = \frac{f}{\omega_0^2 - \omega^2}$$

$\rightarrow$ tells us that the oscillations $x = x_0 e^{i\omega t}$ are in _phase_ with the external force when $\omega^2 < \omega_0^2$ but out of _phase_ when $\omega^2 > \omega_0^2$

$\ast$ Proposed law for phonon-mediated attraction

$\rightarrow$ attractive potential $-V_0$ if $|\varepsilon_{k_i} - \varepsilon_{k_i'}| < \hbar\omega_D$

$\rightarrow$ scattering only possible if $\vec{k}_i'$ is unoccupied

$$V(\vec{k} - \vec{k}') = \begin{cases} -V_0 & \text{if } |\varepsilon_{\vec{k}} - \varepsilon_F| \leq \hbar\omega_D \text{ and } |\varepsilon_{\vec{k}'}| - \varepsilon_F| \leq \hbar\omega_D \\ \\ 0 & \text{otherwise} \end{cases}$$

$2\Delta k$ range of interaction

$$\frac{\Delta k}{k_F} \sim \frac{\hbar\omega_D}{\varepsilon_F} \quad , \quad \varepsilon_F = \frac{\hbar^2 k_F^2}{2m}$$

* In our previous look at the BCS Hamiltonian

$$\hat{H} = \sum_k \begin{pmatrix} c_{k\uparrow}^\dagger & c_{-k\downarrow} \end{pmatrix} \begin{pmatrix} \varepsilon_k - \mu & \Delta \\ \Delta^\dagger & \mu - \varepsilon_k \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c_{-k\downarrow} \end{pmatrix}$$

$$= \sum_k \begin{pmatrix} \gamma_{k1}^\dagger & \gamma_{k2}^\dagger \end{pmatrix} \begin{pmatrix} \sqrt{(\varepsilon_k - \mu)^2 + |\Delta|^2} & 0 \\ 0 & -\sqrt{(\varepsilon_k - \mu)^2 + |\Delta|^2} \end{pmatrix} \begin{pmatrix} \gamma_{k1} \\ \gamma_{k2} \end{pmatrix}$$

$$= \sum_k \sum_{n=\pm} n E_k \gamma_{kn}^\dagger \gamma_{kn}$$

diagonalized energy eigenvalues

Bogoliubov transform

$$E_{k2} = + E_k$$
$$E_{k1} = - E_k$$

$\rightarrow$ gap equation

$$\Delta(k) = \sum_{k'} V(k - k') \Delta(k') \frac{\left(f(E_{k2}) - f(E_{k1}) \right)}{E_{k2}' - E_{k1}'}$$

$$\Delta = \sum{}' \frac{(-V_0)}{2E_k} \left(f(E_k) - f(-E_k) \right) \Delta$$

$$1 = -V_0 \sum{}' \frac{\left(f(E_k) - (1 - f(E_k)) \right)}{2E_k}$$

$$= V_0 \sum{}' \frac{1 - 2f(E_k)}{2E_k}$$

$$1 = V_0 \int_0^{\hbar\omega_D} d\varepsilon \; \frac{D(\varepsilon + \varepsilon_F)}{\sqrt{\varepsilon^2 + \Delta^2}} \left(1 - \frac{2}{e^{E_k/k_B T} + 1} \right)$$

$$\frac{1}{D(\varepsilon_F)V_0} = \int_0^{\hbar\omega_D} \frac{d\varepsilon}{\sqrt{\varepsilon^2 + \Delta^2}} \tanh \sqrt{\frac{\varepsilon^2 + \Delta^2}{2k_B T}}$$

Critical T_c, $\Delta \to 0$

$$\frac{1}{D(\varepsilon_F)V_0} = \int_0^{\hbar\omega_D} \frac{d\varepsilon}{\varepsilon} \tanh \frac{\varepsilon}{2k_B T_c}$$

$$k_B T_c = 1.14 \, \hbar\omega_D \, e^{-1/D(\varepsilon_F)V_0}$$

We already showed that

$$\Delta_0 = 2\hbar\omega_D \, e^{-1/D(\varepsilon_F)V_0}$$

So $\quad 2\Delta_0 = 3.52 \, k_B T_c$

Also,
this provides an explanation for the isotope effect since $\quad \omega_D \sim M^{-1/2}$