Physics 7330: Assignment 1

(to be submitted by Friday, September 4, 2026)

I invite you to attempt Assignment 1 and to submit all your work for Questions 1 and 2. Please turn in a paper copy in class. You're welcome to use Mathematica to help you with any part of the assignment. If you do, please email to `kbeach@olemiss.edu` a single Wolfram Notebook, sent as an attachment, that contains any relevant Mathematica code. Use the naming convention `Phys7330-A1-webid.nb`, and include `Phys7330-Fall2026-webid Assignment 1 Submission` on the subject line. (Be sure to replace `webid` with your own personal UM WebID; mine, for instance, is `kbeach`.) If you end up doing the entire assignment by hand, just turn in a paper copy and ignore the email attachment instructions.

1. The commutator of operators $\hat{A}$ and $\hat{B}$ is defined as $[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$. (Note that version 14.3 of Mathematica introduces the commands `Commutator` and `NonCommutativeExpand`.)

   (a) Show explicitly that $[\hat{A}, \hat{B}] = -[\hat{B}, \hat{A}]$ and $[\hat{A}, \hat{A}] = 0$.

   (b) For operators $\hat{A}$, $\hat{B}$, and $\hat{C}$ and complex numbers α and β, demonstrate that
   $$[\alpha\hat{A} + \beta\hat{B}, \hat{C}] = \alpha[\hat{A}, \hat{C}] + \beta[\hat{B}, \hat{C}].$$

   (c) Prove that the commutator distributes over an operator product as follows:
   $$[\hat{A}, \hat{B}\hat{C}] = [\hat{A}, \hat{B}]\hat{C} + \hat{B}[\hat{A}, \hat{C}].$$

   (d) Prove the famous Jacobi identity:
   $$[\hat{A}, [\hat{B}, \hat{C}]] + [\hat{B}, [\hat{C}, \hat{A}]] + [\hat{C}, [\hat{A}, \hat{B}]] = 0.$$

   (e) Consider operators $\hat{x}$ and $\hat{p}$ satisfying $[\hat{x}, \hat{p}] = i\hbar$. Argue that $[\hat{x}, \hat{p}^2] = 2i\hbar\hat{p}$, $[\hat{x}, \hat{p}^3] = 3i\hbar\hat{p}^2$, and more generally, $[\hat{x}, \hat{p}^n] = ni\hbar\hat{p}^{n-1}$. Use this result to show that
   $$[\hat{x}, f(\hat{p})] = i\hbar f'(\hat{p}),$$
   where f is any smooth function.

2. Consider a one-dimensional quantum system in which position ($\hat{x}$) and linear momentum ($\hat{p}$) are canonically conjugate operators that obey the commutation relation $[\hat{x}, \hat{p}] = i\hbar$. Suppose that state of the system $|\psi\rangle$ is described by a smooth wave function $\langle x|\psi\rangle = \psi(x)$ when represented in the position basis—i.e., the set of position eigenstates $\{|x\rangle : x \in \mathbb{R}\}$ such that $\hat{x}|x\rangle = x|x\rangle$. This framework provides a bridge to the Schödinger language of differential operators acting on wave functions: $\langle x|\hat{x}|\psi\rangle = x\psi(x)$ and $\langle x|\hat{p}|\psi\rangle = (\hbar/i)(d/dx)\psi(x) = -i\hbar\psi'(x)$.

   (a) Prove that $\langle x|\hat{x}\hat{p}|\psi\rangle = x(\hbar/i)\psi'(x)$.

   (b) Prove that $\langle x|\hat{p}\hat{x}|\psi\rangle = x(\hbar/i)\psi'(x) - i\hbar\psi(x)$.

   (c) Compute $\langle x|\hat{p}\hat{x}^2|\psi\rangle$, $\langle x|\hat{x}\hat{p}\hat{x}|\psi\rangle$, and $\langle x|\hat{x}^2\hat{p}|\psi\rangle$.

   (d) Compute $\langle x|\hat{x}\hat{p}^2|\psi\rangle$, $\langle x|\hat{p}\hat{x}\hat{p}|\psi\rangle$, and $\langle x|\hat{p}^2\hat{x}|\psi\rangle$.