Physics 711: Assignment 5 (bonus)

(to be submitted by Wednesday, December 3, 2025)

I invite you to attempt Assignment 5 and to submit all your work for Questions 1 and 2. Please turn in a paper copy in class. Also email to kbeach@olemiss.edu a single Wolfram Notebook, sent as an attachment, that contains any relevant Mathematica code. Use the naming convention `Phys711-A5-webid.nb`, and be sure to include `Phys711-Fall2025-webid Assignment 5 Submission` on the subject line. (Be sure to replace `webid` with your own personal UM WebID; mine, for instance, is `kbeach`.)

Suppose we're given a Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{V} = \sum_{n=0}^{\infty} \epsilon_n |n\rangle\langle n| + \sum_{n=0}^{\infty} \big(\Delta_n |n\rangle\langle n+1| + \Delta_n^* |n+1\rangle\langle n|\big)$$

that describes a hierarchy of energy levels, each coupled to the levels immediately above and below. Let's assume that $\epsilon_n = \epsilon_0 \lfloor (n+3)/2 \rfloor$ and $\Delta_n = (\epsilon_0/2)(i/2)^n$. (With this definition, the ϵ_n are only weakly ordered, and there are duplicate values on the main diagonal.) Assuming that the states $\{|n\rangle\}$ form an orthonormal basis, the Hamiltonian has a matrix description

$$H = \begin{pmatrix} \epsilon_0 & \Delta_0 & 0 & 0 & 0 & \cdots \\ \Delta_0^* & \epsilon_1 & \Delta_1 & 0 & 0 & \\ 0 & \Delta_1^* & \epsilon_2 & \Delta_2 & 0 & \\ 0 & 0 & \Delta_2^* & \epsilon_3 & \Delta_3 & \\ 0 & 0 & 0 & \Delta_3^* & \epsilon_4 & \\ \vdots & & & & & \ddots \end{pmatrix} = \epsilon_0 \begin{pmatrix} 1 & 1/2 & & & & \\ 1/2 & 2 & i/4 & & & \\ & -i/4 & 2 & -1/8 & & \\ & & -1/8 & 3 & -i/16 & \\ & & & +i/16 & 3 & \\ & & & & & \ddots \end{pmatrix}.$$

1. Break the energy degeneracies by diagonalizing within each degenerate 2×2 block, at least for the entries of the 5-column, 5-row snippet shown above.

2. Having completed Question 1, parameterize the transformed Hamiltonian as $\hat{H} = \hat{H}_{\text{diag}} + \lambda \hat{H}_{\text{offdiag}}$. Use nondegenerate perturbation theory to compute the ground-state energy shift $\Delta_0 = E_0 - \epsilon_0$ to order λ^2 around the $\lambda = 0$ solution. You should be able to show that $\Delta_0 = \epsilon_0(1 - 4\lambda^2/15) + O(\lambda)^3$.