Physics 651: Assignment 7

(to be submitted by Tuesday, November 29, 2022)

1. Consider the function $f : \mathbb{C} \to \mathbb{C}$ defined by

$$f(z) = \frac{A}{(z-3)(z+4+i)} + \frac{B}{z(z^2+1)}.$$

   (a) Prove that $f(z)$ is meromorphic by rewriting the function as an explicit sum of five simple poles; i.e.,

$$f(z) = \sum_{k=1}^{5} \frac{C_k}{z - z_k}.$$

   (b) Report the pole location z_k and the residue $\mathrm{Res}(f, z_k)$ for each of $k = 1, 2, \ldots, 5$.

   (c) Evaluate the integral

$$\oint_C dz\, f(z)$$

   around a closed contour C that corresponds to a circle of radius 3/2 centred on the point $z = i/2$, traversed clockwise. Sketch a diagram showing the contour and the location of the poles.

   You may find it helpful to check your answers with *Mathematica*:

```
f[z_] := A/((z - 3) (z + 4 + \[ImaginaryI])) + B/(z (z^2 + 1))
Residue[f[z], {z, 0}]
-ResidueSum[{f[z], Abs[z - \[ImaginaryI]/2] < 3/2},z]
```

   Recall that the imaginary number i can also be produced by typing the combination `[esc]ii[esc]`.

2. (6.11) Evaluate the counterclockwise integral around the circle $|z| = 1$

$$\oint \left(\sinh^2 2z - 4\cosh^3 z\right) \frac{dz}{z}.$$

3. (6.18) Derive the first three terms of the Laurent series for $f(z) = 1/(e^z - 1)$.

4. (6.34) Show that the Heaviside function $\theta(y) = (y + |y|/(2|y|)$ is given by the integral

$$\theta(y) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} dx\, \frac{e^{iyx}}{x - i\epsilon}$$

   in which ϵ is a positive infinitesimal number.