Physics 725: Solid State Physics I

In-class Midterm Exam

Tuesday, October 20, 2015 / 11:00–12:15 / Room 104, Lewis Hall

Student's Name: ____________________

Instructions

There are 16 questions, some with multiple subparts. You should attempt all of them. Mark your response on the test paper in the space provided. **Please use a pen**. If in answering a question you sketch a diagram, please provide meaningful labels. Aids of any kind—including class notes, textbooks, cheat sheets, and calculators—are not permitted.

Good luck!

12 points	multiple choice	questions 1–10
5	short answer	11–14
13	long answer	15–16
30 points		

Multiple choice questions (12 points)

Answer by circling one of (a), (b), (c), etc. directly on the test paper. Be sure that your selection is clear and unambiguous.

1. Which of the following materials is the least ionic in character?
   (a) Ge
   (b) GaAs
   (c) InSb
   (d) InAs
   (e) CdTe
   (f) ZnSe

2. We could arrange a two-dimensional ionic solid in which of the following patterns?
   (a) Kagomé
   (b) triangular
   (c) honeycomb
   (d) none of the them

3. Condensed matter physics largely considers physical processes over what range of energies?
   (a) μeV to eV
   (b) meV to keV
   (c) keV to MeV
   (d) MeV to GeV

4. What is the approximate distance between atoms in a copper crystal?
   (a) 30 Å
   (b) 300 Å
   (c) 0.3 nm
   (d) 30 nm
   (e) 3 μm

(2 points) 5. Which of the following exhibit collective excitations in response to shear?

(yes / no) gas
(yes / no) liquid
(yes / no) glass
(yes / no) regular solid

(2 points) 6. To each description below, assign the most appropriate of the following phonon modes/branches:

(a) acoustic (b) optical (c) longitudinal (d) transverse

Write the corresponding letter in the space provided. Each letter can be appear more than once or not at all.

___ exponentially suppressed contribution to the low-temperature heat capacity
___ T^4 contribution to the low-temperature energy of a three-dimensional crystal
___ atomic motion is perpendicular to the direction of wave propagation
___ can only exist in a crystal with more than one atom per unit cell

7. In its cubic unit cell, $BaTiO_3$ (barium titanate) has barium atoms positioned at the 8 cell corners, a titanium atom in the middle, and oxygen atoms centred at each of the 6 faces. What is the size of the crystal basis?

(a) 3
(b) 4
(c) 5
(d) 8
(e) 15

8. In the linear molecule acetylene (H–C≡C–H), what kind of (carbon valence orbital) hybridization takes place?

(a) sp
(b) sp^2
(c) sp^3

9. At small wave vector $\boldsymbol{q}$, the group velocity and phase velocity necessarily coincide for which one of the following categories of phonon mode?

(a) acoustic
(b) optical
(c) longitudinal
(d) transverse

10. The pair distribution function $g(r) \sim \langle \sum_{i<j} \delta(r - |\boldsymbol{R}_i - \boldsymbol{R}_j|) \rangle$ is a histogram of the observed separation between atoms. For a molten system, it is flat and featureless—aside from a small hard-core exclusion region $0 \leq r < \sigma$. When the system is cooled and solidifies, $g(r)$ develops a sequence of strong peaks at positions $r > \sigma$. For each sequence below (showing the first three peak distances), identify the Bravais lattice type as

(a) body-centred cubic, (b) face-centred cubic, or (c) simple cubic.

Write the corresponding letter in the space provided. Each letter should appear exactly once.

___ $\frac{1}{\sqrt{2}}a,\ a,\ \frac{\sqrt{10}}{2}a, \ldots$
___ $a,\ \sqrt{2}a,\ \sqrt{3}a, \ldots$
___ $\frac{\sqrt{3}}{2}a,\ a,\ \frac{\sqrt{11}}{2}a, \ldots$

Short answer questions (5 points)

Try to provide answers in concise prose. At most a few sentences are required for each question.

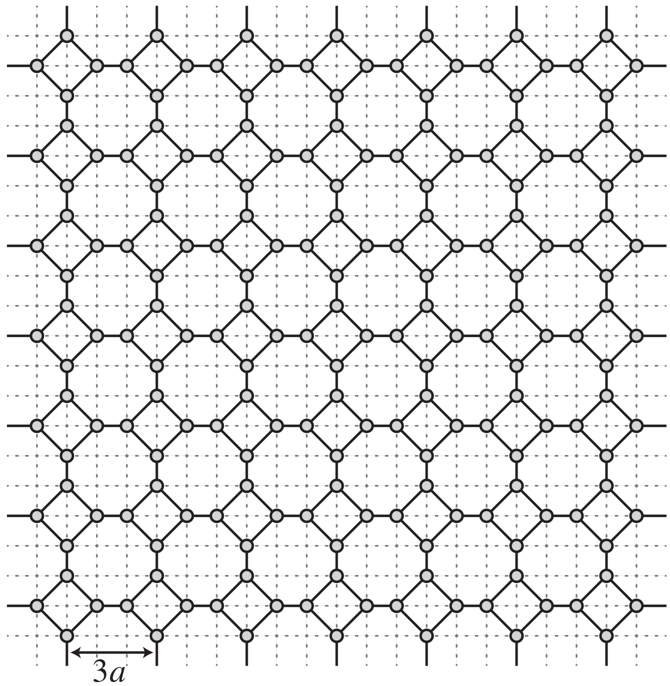
11. Two regions of a crystal that have grown out from far separated nucleation centres are generically incompatible and meet along a surface of mismatches. What do we call this surface?

(2 points) 12. What are x-rays? What energy or wavelength must they have if they're to be used in diffraction experiments for the purpose of characterizing crystal structure? When x-rays scatter from a material, are they interacting with protons, neutrons, or electrons?

13. What is the Born-Oppenheimer approximation?

14. The Madelung constant of a stable ionic solid is always positive. Why must that be so?

Long answer problems (13 points)



15. The figure above shows a two-dimensional crystal made up of a single kind of atom. The underlying grid of dotted lines is just a guide to the eye. (The dotted lines are spaced by a distance a.)

(2 points) (a) Find lattice vectors and a basis for the crystal.

(b) Compute the area of the unit cell.

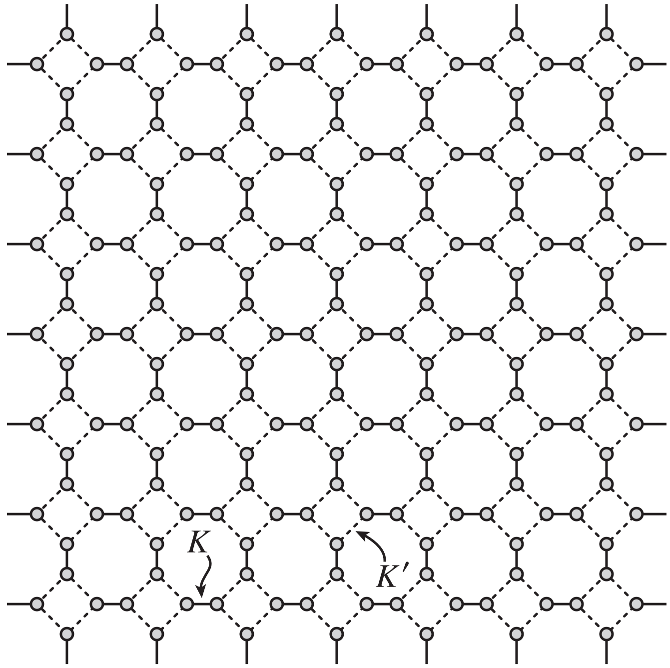
(c) Determine the corresponding reciprocal lattice vectors.

(d) Sketch the Brillouin zone that results from the Wigner-Seitz construction.

(3 points) (e) Using the reciprocal lattice vectors $\boldsymbol{g}_1$ and $\boldsymbol{g}_2$ that you computed in part (c), parameterize arbitrary points in the reciprocal lattice with the linear combination

$$\boldsymbol{G} = \boldsymbol{G}(n_1, n_2) = n_1 \boldsymbol{g}_1 + n_2 \boldsymbol{g}_2$$

for integers n_1 and n_2. Assuming that the atomic form factors are identical for all the atoms, determine the geometric structure factor as a function of n_1 and n_2.



16. Here's the same lattice from the previous question, where the lines now represent two types of covalent bond. The solid lines denote bonds of equilibrium length a and Hooke's law spring constant K; the dashed lines denote bonds of length $\sqrt{2}a$ and spring constant K'.

    The total vibrational energy (kinetic plus potential) is

$$E_{\text{vib}} = \sum_i \frac{m}{2}(\dot{\boldsymbol{r}}_i)^2 + \frac{K}{2}\sum_{\langle i,j\rangle}\Big(|\boldsymbol{r}_j - \boldsymbol{r}_i| - a\Big)^2 + \frac{K'}{2}\sum_{\langle\langle i,j\rangle\rangle}\Big(|\boldsymbol{r}_j - \boldsymbol{r}_i| - \sqrt{2}a\Big)^2,$$

    where $\langle i, j\rangle$ and $\langle\langle i, j\rangle\rangle$ denote neighbouring sites a distance a and $\sqrt{2}a$ apart, respectively.

    (a) The phonon dispersion relation can be expressed in compact form as an eigenproblem, $\omega^2\xi = D\xi$, where each of ω, ξ, and D are functions of a wave vector $\boldsymbol{q} = (q_x, q_y)$ in the Brillouin zone. The polarization ξ is an N-component vector, and the dynamical matrix D is an $N \times N$ matrix. What is the value of N?

    (b) Explain why there are six optical modes.

(c) The eigenvector $\xi^{(\lambda)}$ describes the pattern of motion oscillating with angular frequency $\omega_{\boldsymbol{q}}^{(\lambda)}$ in each modes $\lambda = 1, 2, \ldots, N$. Draw the two for which $\lim_{\boldsymbol{q}\to 0} \omega_{\boldsymbol{q}}^{(\lambda)} = 0$.

(d) Drawn the pattern that corresponds to the optical mode with highest energy.

(e) Consider the limit where $K' \gg K > 0$. (This should remind you of the square-lattice with bond-directed forces example we did in class.) Does this system support longitudinal acoustic modes? Does this system support transverse acoustic modes?

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