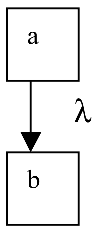
RADIOACTIVE DECAY CHAINS **PHYS 415/417**

Units of Activity: *1Bq (Becquerel) = dps = 1 decay/s* *1 Ci (Curie) = 3.7e10dps*

I , a→ b Consider the decay of a *parent* radioactive nuclei from state **a** to stable *daughter* **b** with decay rate λ. The activity of a source *A* refers to the number of decays per second or rate of decay.



Exponential Decay Law

Our solution is just the exponential decay law.

$dN/N = -\lambda\ dt$

$\mathbf{N(t) = N(0) \exp\{-\lambda t\}}$ (1)

The average lifetime $\boldsymbol{\tau = 1/\lambda}$

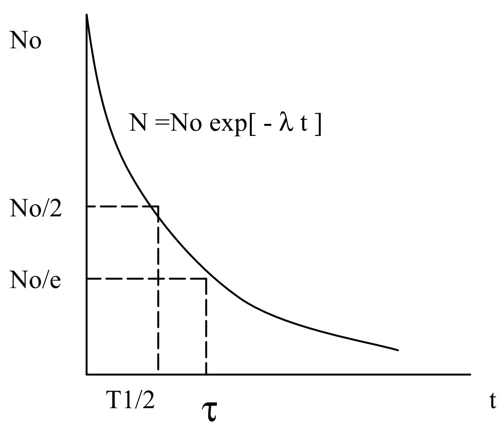
The half life is $\mathbf{T_{1/2} = 0.693\,\tau}$ (time for N→ N/2)

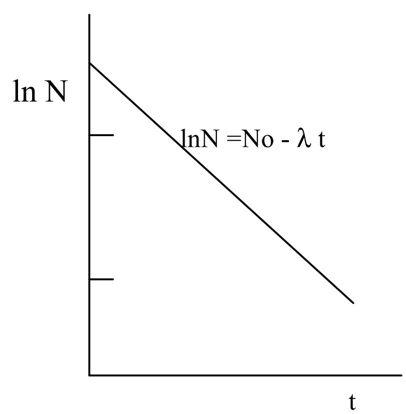
$N(t)/N(0) = 1/2 = \exp(-\lambda t_{1/2}) \rightarrow 0.693 = -\lambda t_{1/2} \rightarrow t_{1/2} = 0.693/\lambda = 0.693\,\tau$

Differentiating (1) $dN/dt = \lambda N \exp\{-\lambda t\}$

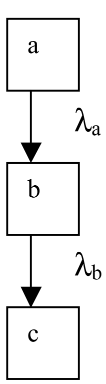
The activity *A* of the sample is also dropping $\boldsymbol{A(t) = N(0)\,\lambda\ \exp\{-\lambda a\, t\}}$ (2)

From (2) we see that the activity at $\mathbf{A(t) = \lambda N(t)}$





II. a→ b→ c Now consider a double decay chain in which a *parent* nuclei **a** decays to *daughter* **b**, and daughter b decays to *stable nuclei* **c**.



<u>**General Case**</u> $N_b(0) = 0$

$dN_a/dt = -\lambda_a N_a$ (3)

$dN_b/dt = -\lambda_b N_b + \lambda_a N_a$ (4)

$N_a = N_a(0) \exp((-\lambda_a t)$ (5)

$$N_b = \frac{\lambda_a}{\lambda_b - \lambda_a} N_a(0) [\exp(-\lambda_a t) - \exp(-\lambda_b t)] \quad (6)$$

Secular Equilibrium $T_a >> T_b$ ($\lambda_a << \lambda_b$) (*Equilibrium in time*) The parent has a very long half-life T_a with repect to daughter T_b**.** The parent activity A_a is nearly constant because it has a very long half-life (about all naturally occurring radionuclides).

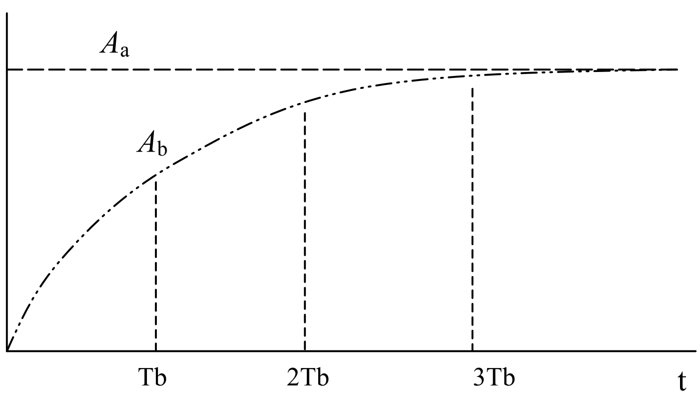
From (4) $dNb / [-\lambda b\ Nb + A_a] = dt$ *Letting* $\lambda a\ Na = A_a = A_a(0) = constant$

Let $u = -\lambda b\ Nb + Aa$ and $du = -\lambda b\ dNb$

$du/u = -\lambda b\ dt$

$u = uo \exp[-\lambda b\ t]$ or $A_a - \lambda_b N_b = [A_a - \lambda_b N_b(0)] \exp[-\lambda_b t]$

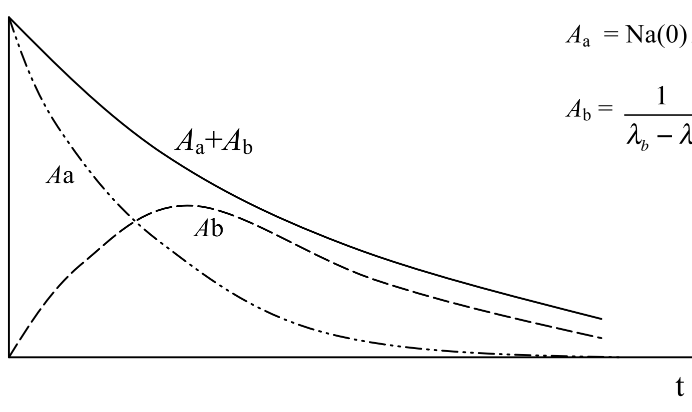
$$\mathbf{A_a = A_a(0)} \quad (7)$$
$$\mathbf{A_b = A_a(0)\,[1 - \exp(-\lambda_b t)] + A_b(0) \exp(-\lambda_b t)} \quad (8)$$



Secular Equilibrium-
The daugter activity reaches equilibrium with the parent activity after ~3 T_b daughter halflives, In this plot $A_b(0) = 0$ or no daughters present at t=0.

Transient Equlibrium Ta > Tb Parent decay rate is somewhat larger.

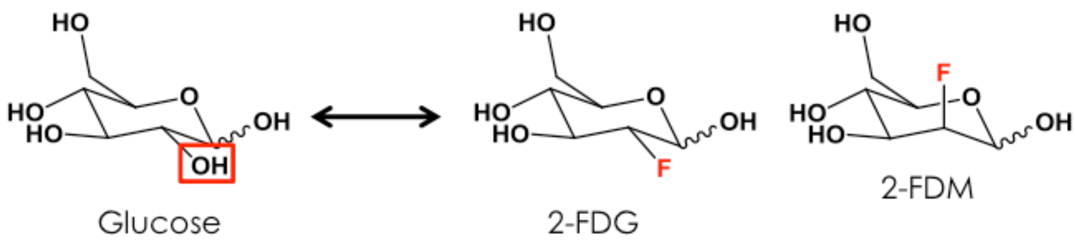
Daughter activity builds up and then starts to drop in unison with parent. They reach a so-called *transient equilibrium*. F-18 ($T_{1/2}$=110 min)



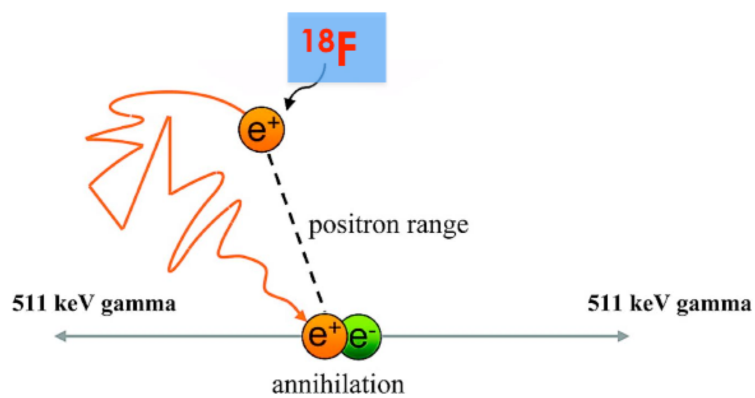
$A_a = Na(0)\,\lambda_a \exp((-\lambda_a t)$ from (5)

$$A_b = \frac{1}{\lambda_b - \lambda_a} A_a(0)\,[\lambda_a \exp(-\lambda_a t) - \lambda_b \exp(-\lambda_b t)]$$

Fluorine-18 is an important isotope in the radiopharmaceutical industry, and is primarily synthesized into fluorodeoxyglucose (FDG) for use in positron emission tomography (PET scans).



${}_9F^{18}$ -> ${}_8O^{18}$ + ${}_{+}e$ ${}_{+\,v}$ Inverse Beta decay then Positron Annihilation $e_+ + e- \rightarrow \gamma\gamma$



Questions to answer.

1- What is the value for the decay constant λ of Cs-137 ($T_{1/2}$=30.17 y) ?

2- The isotope I^{132} decays by beta emission in to stable Xe^{132} with a halflife of 2.3 h.
(a) How long will it take for 1/4 of the original I^{132} atoms to decay?
(b) How long will it take for the I^{132} sample to loose 60% of its activity?

3- How many atoms are there in a 1 MBq source of Cs-137?

4- A 10 mg sample of Sr^{90} (T=29.12y) is in secular equilibrium with its daughter Y^{90}.
(a) How many Bq of Sr^{90} is present ?
(b) How many Bq of Y^{90} is present ?
(c) What is the mass of Y^{90} present?
(d) What will be the activity of Y^{90} after 100 y?