Chapter  The Nature of Light

Propagaton of Light
The Electromagnetic Spectrum
Maxwell’s description of the EM Wave
Intensity and the Poynting Vector

Reflection and Refraction of Light
Total Internal Reflection
Polarization and Mallus’s Law

Today’s information age is based almost entirely on the physics of electromagnetic waves. The connection between electric and magnetic fields to produce light is one of the greatest achievements produced by physics, and electromagnetic waves are at the core of many fields in science and engineering.

In this chapter we introduce fundamental concepts and explore the properties of electromagnetic waves.
Propagation of Light

Light Rays
- Rectilinear Propagation
- Supported particle nature of light
- Intensity \( I(x) = I_0 \) (W/m²)

Point Sources
- Inverse square law
- Intensity \( I(r) = I_0 \frac{1}{4\pi r^2} \)

\[ I = I_0 \left( \frac{W}{m^2} \right) \]

\[ I = I_0 \frac{1}{4\pi r^2} \]
Electromagnetic Spectrum - Newton’s Rainbow (1671)

The wavelength/frequency range in which electromagnetic (EM) waves (light) are visible is only a tiny fraction of the entire electromagnetic spectrum.
Maxwell (1865) formulates a theory of light as an Electric + Magnetic wave traveling with the speed $c = 3 \times 10^8$ m/s.

- We say the wave is polarized along Electric field direction.

The Poynting vector describes the energy flux of the EM or energy transfer. It has units of energy flux (W/m$^2$) or Power/area.

\[ \mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \quad \text{energy flux} \]

\[ I(W/m^2) = |\mathbf{S}| = \frac{E_0 B_0}{\mu_0} = \frac{E_0^2}{\mu_0 c} \]
Mathematical Description of Traveling EM Waves

Electric Field: \( E = E_m \sin(kx - \omega t) \)

Magnetic Field: \( B = B_m \sin(kx - \omega t) \)

Wave Speed: \( c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \)

All EM waves travel at a speed \( c \) in vacuum.

Angular Frequency: \( \omega = \frac{2\pi}{T} \)

Wave Number: \( k = \frac{2\pi}{\lambda} \)

\( \nu_c = c = \frac{\omega}{k} = \frac{\lambda}{T} \)

\( \vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \) \( \rightarrow \) \( E = cB \)

\( I = \frac{1}{\mu_0 c} \left| \vec{E} \right|^2 \)
The Traveling Electromagnetic (EM) Wave, Qualitatively

An LC oscillator causes currents to flow sinusoidally, which in turn produces oscillating electric and magnetic fields, which then propagate through space as EM waves.

Oscillation Frequency:

\[ \omega = \sqrt{\frac{1}{LC}} \]

Circular wave pattern from a line source:

\[ E \sim \frac{\lambda}{2\pi \varepsilon_0 r} \]

\[ B \sim \frac{\mu_0 i}{2\pi r} \]

Plane wave
Q1: A laser beam delivers 50 mW/mm$^2$ to a target. Find the 
value of the electric and magnetic field amplitudes of the 
light wave.

\[ I = \frac{P}{A} = 50 \times 10^{-3} \frac{W}{\text{mm}^2} \]

\[ I = 0.05 \frac{W}{m^2} \]

\[ I = |\vec{S}| = \left( \frac{1}{\mu_0} \right) E \times B \]

\[ \left( \frac{1}{\mu_0} \right) E B \]

\[ Using \ E = cB \rightarrow I = \left( \frac{1}{\mu_0} \right) E B \rightarrow E^2 = \mu_0 c I \]

\[ E = \left( 4\pi \times 10^{-7} \right) \left( 3 \times 10^8 \right) \left( 0.05 \right) \]

\[ E_{\text{rms}} = 18.8 \frac{V}{m} \]

\[ B = E / c = \left( \frac{18.8}{3 \times 10^8} \right) \]

\[ B_{\text{rms}} = 6.3 \times 10^{-8} T \]

Q2: An observer is 1.8m from a small light source (candle) 
whose power output is 250W. Calculate the value of the values 
of the E and B fields at the position of the observer.

\[ I = I_0 / 4\pi r^2 \quad \text{intensity/area of spherical wavefront at } r = 1.8m \]

\[ I = 250W / 4\pi r^2 = 250 / 4\pi (1.8m)^2 \]

\[ I = 6.2 \frac{W}{m^2} \]

\[ E^2 = \mu_0 c I \]

\[ 2315 \left( V^2 / m^2 \right) \]

\[ E_{\text{rms}} = 48.1 \frac{V}{m} \]

\[ B = E / c \]

\[ B_{\text{rms}} = 1.6 \times 10^{-7} T \]
Reflection and Refraction

In a vacuum light will travel in a straight line. When light encounters a medium it will either reflect or refract (transmit with bend). The velocity of light in a medium of index of refraction $n$ is $v_n = c / n$.

What happens when a narrow beam of light encounters a glass surface?

**Law of Reflection**

Reflection: $\theta_1 = \theta_1'$

**Snell’s Law**

Refraction: $n_1 \sin \theta_1 = n_2 \sin \theta_2$

$n$ is the index of refraction.
The **refractive index** can be seen as the factor by which the speed and the wavelength of the radiation are reduced with respect to their vacuum values: the speed of light in a medium is \( v = \frac{c}{n} \), and similarly the wavelength in that medium is \( \lambda = \frac{\lambda_0}{n} \), where \( \lambda_0 \) is the wavelength of that light in vacuum.

\[
v_c = f \lambda_0 \rightarrow f \frac{\lambda_0}{n} = \frac{f \lambda_0}{n} \quad \Rightarrow \quad v_c = \frac{c}{n}
\]

\[
\lambda = \frac{\lambda_0}{n} \quad f = f_0
\]
For light going from media $n_1$ to $n_2$

(a) $n_2 = n_1$  \( \theta_{12} = \theta_1 \)

(b) $n_2 > n_1$  \( \theta_2 < \theta_1 \), light bent towards normal

(c) $n_2 < n_1$  \( \theta_2 > \theta_1 \), light bent away from normal
How do metals and other solid materials reflect light?

In the case of metals the free electrons in the surface layers deflect the light elastically with, $\theta_1 = \theta_2$, (specular reflection). Other materials will absorb certain colors and reflect others (diffuse reflection), giving the material its color.

Specular reflection

Diffuse reflection

I. Regular Reflection

II. Irregular Reflection

8: Reflection from smooth and rough surfaces.
Indexes of refraction of Some Materials

<table>
<thead>
<tr>
<th>Medium</th>
<th>Red (660 nm)</th>
<th>Orange (610 nm)</th>
<th>Yellow (580 nm)</th>
<th>Green (550 nm)</th>
<th>Blue (470 nm)</th>
<th>Violet (410 nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1.331</td>
<td>1.332</td>
<td>1.333</td>
<td>1.335</td>
<td>1.338</td>
<td>1.342</td>
</tr>
<tr>
<td>Diamond</td>
<td>2.410</td>
<td>2.415</td>
<td>2.417</td>
<td>2.426</td>
<td>2.444</td>
<td>2.458</td>
</tr>
<tr>
<td>Glass, crown</td>
<td>1.512</td>
<td>1.514</td>
<td>1.518</td>
<td>1.519</td>
<td>1.524</td>
<td>1.530</td>
</tr>
<tr>
<td>Glass, flint</td>
<td>1.662</td>
<td>1.665</td>
<td>1.667</td>
<td>1.674</td>
<td>1.684</td>
<td>1.698</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>1.488</td>
<td>1.490</td>
<td>1.492</td>
<td>1.493</td>
<td>1.499</td>
<td>1.506</td>
</tr>
<tr>
<td>Quartz, fused</td>
<td>1.455</td>
<td>1.456</td>
<td>1.458</td>
<td>1.459</td>
<td>1.462</td>
<td>1.468</td>
</tr>
</tbody>
</table>

*Table 1.2 Index of Refraction *n* in Selected Media at Various Wavelengths*
Q3: A fisherman observes a fish at viewing angle $\phi = 12^0$. He believes the fish to be 1m below the surface of the water. Determine $x_0$ and then determine the true depth $y_0$ of the fish?

\[
\begin{align*}
\tan \phi &= 1m / x_0 \\
x_0 &= 1m / \tan(12^0) = 4.7 m \\
\theta_1 &= 90^0 - 12^0 = 78^0 \\
n_1 \sin(\theta_1) &= n_2 \sin(\theta_2) \\
(1.00)\sin(78^0) &= (1.33)\sin(\theta_2) \\
0.978 &= 1.33 \sin(\theta_2) \\
\theta_2 &= \sin^{-1}(0.978 / 1.33) \\
&= 0.826 \text{rad} = 47.3^0 \\
\tan(\theta_2) &= x_0 / y_0 = 1.08 \\
y_0 &= 4.7m / 1.08 = 4.4m
\end{align*}
\]

Q4: What percent of light is reflected and transmitted (at normal incidence) at an air-water interface?

\[
R = \left| \frac{n_w - n_{air}}{n_w + n_{air}} \right|^2 = \left| \frac{1.33 - 1}{1.33 + 1} \right|^2 \\
R = 0.02 \text{(2%)} \quad T = 98% 
\]
Total Internal Reflection

For light that travels from a medium with a larger index of refraction to a medium with a smaller medium of refraction $n_1 > n_1$ and $\theta_2 > \theta_1$, as $\theta_1$ increases, $\theta_2$ will reach $90^\circ$ (the largest possible angle for refraction) before $\theta_1$ does.

Total internal reflection can be used, for example, to guide/contain light along an optical fiber

$$n_1 \sin \theta_c = n_2 \sin 90^\circ = n_2$$

Critical Angle: $\theta_c = \sin^{-1} \frac{n_2}{n_1}$

When $\theta_2 > \theta_c$ no light is refracted (Snell’s Law does not have a solution!) so no light is transmitted

Total Internal Reflection
Fiber Optics are used to transmit signals with low noise and low signal loss.

Figure 1.15 Light entering a thin optic fiber may strike the inside surface at large or grazing angles and is completely reflected if these angles exceed the critical angle. Such rays continue down the fiber, even following it around corners, since the angles of reflection and incidence remain large.
Retro reflectors

Total internal reflection right angle prism.

Retro reflective tape

A rainbow is caused by sunlight and atmospheric conditions. Light enters a water droplet, slowing down and bending as it goes from air to denser water. The light reflects off the inside of the droplet, separating into its component wavelengths—colors. When light exits the droplet, it makes a rainbow.

Corner reflector

\ corner cube retroreflector reflect... esearchgate.net
Refraction by lenses

Spherical lenses made of glass, $n=1.52$, can focus or defocus light by the laws of refraction.

Rays approaching a converging lens from $\infty$ will converge to a focal point.
Thin-Lens Equation: Cartesian Convention

If the Cartesian sign convention is used, the Gaussian form of the lens equation becomes

\[
\frac{1}{o} + \frac{1}{f} = \frac{1}{i}
\]

because in that convention the positive direction for a quantity is always in the direction that light is traveling. The object distance \(o\) is then a negative number because to travel from the lens to the object, you must travel in the direction opposite to light travel. The thin lens equation is also sometimes expressed in the Newtonian form. The derivation of the Gaussian form proceeds from triangle geometry. For a thin lens, the lens power \(P\) is the sum of the surface powers. For thicker lenses, Gullstrand's equation can be used to get the equivalent power.

To common form of lens equation in introductory texts
The polarization of light describes how the electric field in the EM wave oscillates. This wave is vertically or linearly polarized.

Unpolarized or randomly polarized light has its instantaneous polarization direction vary randomly with time.

One can produce different polarizations such as circularly or elliptically polarized light by mixing two linearly polarized beams.
Polarizing Sheet

Only electric field component along polarizing direction of polarizing sheet is passed (transmitted), the perpendicular component is blocked (absorbed).

\[ I = \frac{1}{2} I_0 \]

50% transmission of unpolarized light
Transmission Polarized Light through a polarizer

- Since only the component of the incident electric field $E$ parallel to the polarizing axis is transmitted, $E_{\text{transmitted}} = E_y = E \cos \theta$
- **Malus's Law** states that the intensity of plane-polarized light that passes through an analyzer varies as the square of the cosine of the angle between the plane of the polarizer and the transmission axes of the analyzer.

$$I \sim |E_y|^2$$

$$I = I_0 \cos^2 \theta$$

- For unpolarized light impinging on a polarizer $I = I_0 \int_0^{2\pi} \cos^2 \theta \, d\theta = \frac{1}{2} I_0$

$$I = \frac{1}{2} I_0$$
Q5: Three polarizing filters are oriented with their axis at 30° with respect to each other. θ₁=0°, θ₂=30°, θ₃=60°. Determine the percent transmission of an unpolarized light source through the three filters.

\[ I = \left( \frac{I_0}{2} \right) \cos^2(\Delta \theta_{12}) \cos^2(\Delta \theta_{23}) \]

\[ = \left( \frac{I_0}{2} \right) \cos^2(30°) \cos^2(30°) \]

\[ = \left( \frac{I_0}{2} \right)(0.866)^2(0.866)^2 = 0.28I_0 \]

[28%]
Polarization by Reflection

Upon reflection from a surface only a fraction vertical component of the E-field is reflected, resulting in a net horizontal polarization of the reflected light. The angle of perfect polarization is called Brewster’s Angle. Polaroid sun glasses with a vertical polarization axis will cancel the reflected light (glare) sharpening the image.

\[
\theta_B = \tan^{-1}\left(\frac{n_{\text{media}}}{n_{\text{air}}}\right)
\]

In which direction does light reflecting off a lake tend to be polarized?
STOP HERE
Radiation Pressure

EM waves have linear momentum as well as energy. Light can exert pressure.

\[ \vec{S}_{\text{incident}} \rightarrow \Delta p \]
\[ \vec{S}_{\text{incident}} \rightarrow \Delta p \]
\[ \vec{S}_{\text{reflected}} \rightarrow \Delta p \]

Total absorption:
\[ \Delta p = \frac{\Delta U}{c} \]
\[ p_r = \frac{I}{c} \]

Total reflection Back along path:
\[ \Delta p = \frac{2\Delta U}{c} \]
\[ p_r = \frac{2I}{c} \]

\[ F = \frac{\Delta p}{\Delta t} \]
\[ I = \frac{\text{power}}{\text{area}} = \frac{\text{energy/time}}{\text{area}} \]
\[ = \frac{\Delta U/\Delta t}{A} \]
\[ \Delta U = IA \Delta t \]
\[ F = \frac{IA}{c} \] (total absorption)
\[ F = \frac{2IA}{c} \] (total reflection back along path)
\[ p_r = \frac{F}{A} \]
Radiation Pressure
The index of refraction $n$ encountered by light in any medium except vacuum depends on the wavelength of the light. So if light consisting of different wavelengths enters a material, the different wavelengths will be refracted differently. Chromatic dispersion can be good (e.g., used to analyze wavelength composition of light) or bad (e.g., chromatic aberration in lenses).
Rainbows

Sunlight consists of all visible colors and water is dispersive, so when sunlight is refracted as it enters water droplets, is reflected off the back surface, and again is refracted as it exits the water drops, the range of angles for the exiting ray will depend on the color of the ray. Since blue is refracted more strongly than red, only droplets that are closer the the rainbow center ($A$) will refract/reflect blue light to the observer ($O$). Droplets at larger angles will still refract/reflect red light to the observer.

What happens for rays that reflect twice off the back surfaces of the droplets?

*Fig. 33-22*