Experiment 3: Vector Addition

Figure 3.1: Force Table

EQUIPMENT

Force Table
(4) Pulleys
(4) Mass Hangers
Masses
Level (TA's Table)
(2) Protractors
(2) Rulers
(4) Colored Pencils (bold colors)

Advance Reading

Text: Motion in one and two dimensions, vectors and vector addition.

Objective

The objective of this lab is add vectors using both the tail-to-head method and the component method and to verify the results using a force table.

Theory

A scalar quantity is a number that has only a magnitude. When scalar quantities are added together (e.g., prices), the result is a sum.

Vectors are quantities that have both magnitude and direction; specific methods of addition are required. When vector quantities are added, the result is a **resultant**.

For example, if you walk 1 mile north, then 1 mile east, you will walk a total distance of 2 miles (distance is a scalar quantity). Displacement, a vector, involves both distance and direction. So the same 2 mile walk results in a displacement of $\sqrt{2}$ miles northeast of where you began ($\approx$ 1.41 miles, northeast of your starting position).

A negative vector has the same length as the corresponding positive vector, but with the opposite direction. Making a vector negative can be accomplished either by changing the sign of the magnitude or by simply adjusting the direction by 180°.

$$\vec{\mathbf{V}} = 5\ \text{N}\ \ 100^\circ \qquad -\vec{\mathbf{V}} = -5\ \text{N}\ \ 100^\circ$$

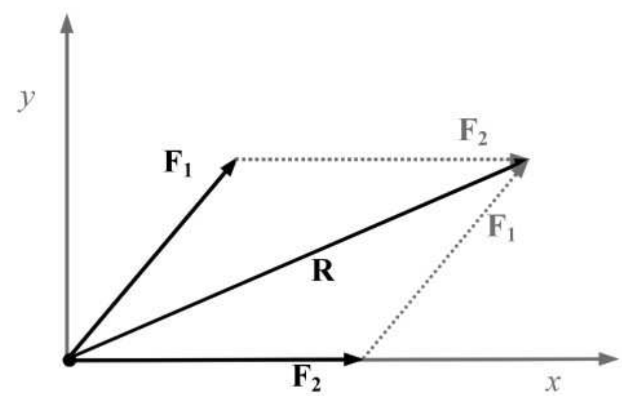
or

$$-\vec{\mathbf{V}} = 5\ \text{N}\ \ 280^\circ$$

Tail-to-Head Method

Vectors can be added together graphically by drawing them end-to-end. A vector can be moved to any location; so long as its magnitude and orientation are not changed, it remains the same vector. When adding vectors, the order in which the vectors are added does not change the resultant.

- Draw each vector on a coordinate system; begin each from the origin.
- Choose any vector drawn to be the first vector.
- Choose a second vector and redraw it, beginning from the end of the first.
- Repeat, adding as many vectors as are desired to the end of the "train" of vectors.
- The resultant is a vector that begins at the origin and ends at the tip of the last vector drawn. It is the shortest distance between the beginning and the end of the path created.



$$\vec{\mathbf{F}}_\mathbf{1} + \vec{\mathbf{F}}_\mathbf{2} = \vec{\mathbf{F}}_\mathbf{2} + \vec{\mathbf{F}}_\mathbf{1} = \vec{\mathbf{R}}$$

Figure 3.2: Adding 2 Vectors, Tail-to-Head

The tail-to-head method is often useful when working problems. A quick sketch, rather than measurements, can help verify your solutions.

Component Method

To add vectors by components, calculate how far each vector extends in each dimension. The lengths of the x- and y-*components* of a vector depend on the length of the vector and the sine or cosine of its direction, θ:

$$sin\theta = \frac{F_{1_y}}{F_1} \qquad cos\theta = \frac{F_{1_x}}{F_1}$$

Use algebra to solve for each component, F_{1_x} and F_{1_y}, from these equations.

$$F_{1_x} = |\vec{\mathbf{F}}_\mathbf{1}|cos\theta \tag{3.1}$$

$$F_{1_y} = |\vec{\mathbf{F}}_\mathbf{1}|sin\theta \tag{3.2}$$

$$\theta = tan^{-1}\left(\frac{F_{1_y}}{F_{1_x}}\right) \tag{3.3}$$

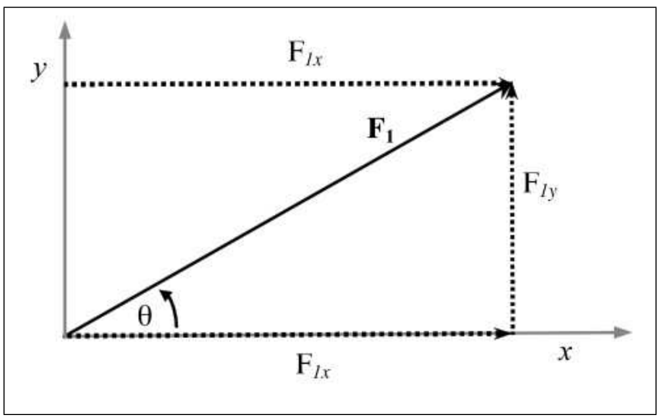


Figure 3.3

When each vector is broken into components, add the x-*components* of each vector:

$$\sum_{i=1}^{n} F_{i_x} = R_x \tag{3.4}$$

Then add all of the y-*components*:

$$\sum_{i=1}^{n} F_{i_y} = R_y \tag{3.5}$$

The sums are the x- and y-*components* of the resultant vector, $\vec{\mathbf{R}}$.

The components of $\vec{\mathbf{R}}$ can be converted back into polar form (R, θ) using the Pythagorean theorem (Eq. 3.6) and the tangent function (Eq. 3.3):

$$|\vec{\mathbf{R}}| = R = \sqrt{R_x^2 + R_y^2} \tag{3.6}$$

$$\theta = tan^{-1}\left(\frac{R_y}{R_x}\right) \tag{3.7}$$

Note: Verify the quadrant! A calculator will return only one of two possible angles (Eq. 3.7). To verify the quadrant, determine if R_x, R_y are positive or negative. If your calculation puts the resultant in quadrant I, but R_x and R_y are both negative, it must be in quadrant III; simply add 180° to the angle.

Force Table Verification

We will use a *force table* to verify our results of vector addition and gain a hands-on perspective. The force table is a circular steel disc with angles 0° to 360° inscribed on the edge (refer to Figure 3.1).

As noted above, when adding vectors, a resultant vector is determined. To balance the force table, however, a force that is equal in magnitude but opposite in direction must be used. This force is the **equilibrant**, $\vec{\mathbf{E}}$. $\vec{\mathbf{E}} = -\vec{\mathbf{R}}$.

For example, when a 10.0 N force at 0° and a 10.0 N force at 90° are added, the resultant force has a magnitude of 14.1 N at 45°. The equilibrant force has the same magnitude, but the direction is 180°+45° = 225°. The equilibrant must be used to balance the two 10.0 N forces.

Name: ____

1. What is a vector? (10 pts)

2. Name a vector quantity and its magnitude. (10 pts)

3. What is a scalar quantity? Give two examples. (10 pts)

4. What is the *equilibrant*? (10 pts)

For *Questions 5, 6,* and *7,* use the following values:

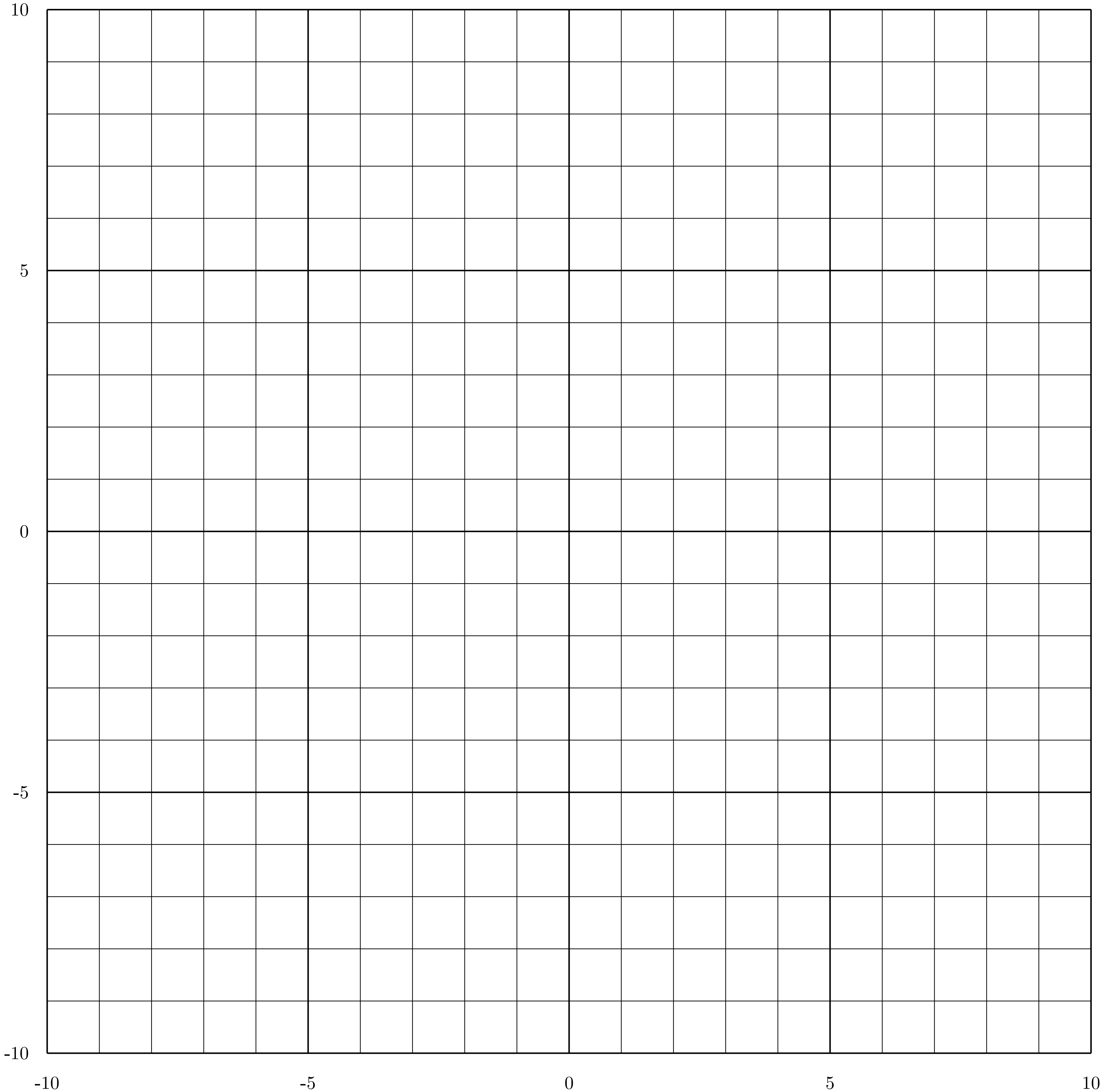
$\vec{\mathbf{A}} = 5.0$ N at $135.0°$ $\qquad$ $\vec{\mathbf{B}} = 6.0$ N at $270.0°$

If you get the same answer for *Questions 5* and *6*, ask for help!

5. Using the component method, add vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ (*i.e.*, $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$). (20 pts)

6. Using the component method, add vectors $\vec{\mathbf{A}}$ and $-\vec{\mathbf{B}}$ (*i.e.*, $\vec{\mathbf{R}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$). (20 pts)

7. Using the Tail-to-Head method, add vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ ($\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$), on the back of this sheet; rulers and protractors will be provided in the physics building. Let 2.0 cm = 1.0 N. (20 pts)


10
5
0
-5
-10
-10
-5
0
5
10

PROCEDURE

PART 1: Tail-to-Head Method

1. Your TA will provide you with a set of three force vectors, record them in the table to the right. Let 1.00 N = 2.00 cm on graph paper.

2. Using a ruler and a protractor, draw the three vectors on the graph paper provided, starting each one from the origin. You should place the origin near the center of the page. Use different colored pencils for each vector.

3. Label the three force vectors.

4. Add together vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ graphically: Draw vector $\vec{\mathbf{B}}$ again, beginning from the tip (or "head") of vector $\vec{\mathbf{A}}$.

5. Next, add the third vector, $\vec{\mathbf{C}}$, to the first two: Redraw it, beginning from the tip of $\vec{\mathbf{B}}$.

6. Draw the resultant, $\vec{\mathbf{R}}$, from the origin to the tip of the last vector drawn, $\vec{\mathbf{C}}$. When the three forces $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ act together, they behave as though they were only one force, $\vec{\mathbf{R}}$.

7. Measure and record $\vec{\mathbf{R}}$. Include uncertainty in your measurement of $|\vec{\mathbf{R}}|$ and θ.

8. Record the equilibrant that balances the three forces. Note that this has the same magnitude as $\vec{\mathbf{R}}$.

PART 2: Force Table

9. Use the level to level the force table.

10. Set three pulleys on the force table in the magnitude and direction of $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$. Note: the mass hanger has its own mass. Let 1.00 N = 100 g on the force table.

11. Add a fourth vector to equalize the forces. This equilibrant force can have any magnitude and direction; you may use your equilibrant from *Step 8* as a guide.

12. Have your TA pull the pin from your force table to see if $\Sigma F = 0$.

13. If the forces are unbalanced, adjust the magnitude and direction of the equilibrant. When they are balanced, the fourth vector is your experimental equilibrant.

14. Calculate your accuracy against the theoretical equilibrant. Show your work.

PART 3: Vector Subtraction

15. Vectors $\vec{\mathbf{D}}$ and $\vec{\mathbf{E}}$ are given by your TA. Record them in the table provided.

16. Calculate the resultant, $\vec{\mathbf{D}} - \vec{\mathbf{E}}$. It may help you to draw a sketch.

17. Find the equilibrant.

QUESTIONS

1. Compare the two methods for $\vec{\mathbf{R}}$ (R and θ) for each set of vectors.

2. Consider six vectors that are added tail-to-head, ending up where they started from. What is the magnitude of $\vec{\mathbf{R}}$?