Experiment 7: Conservation of Energy and Linear Momentum

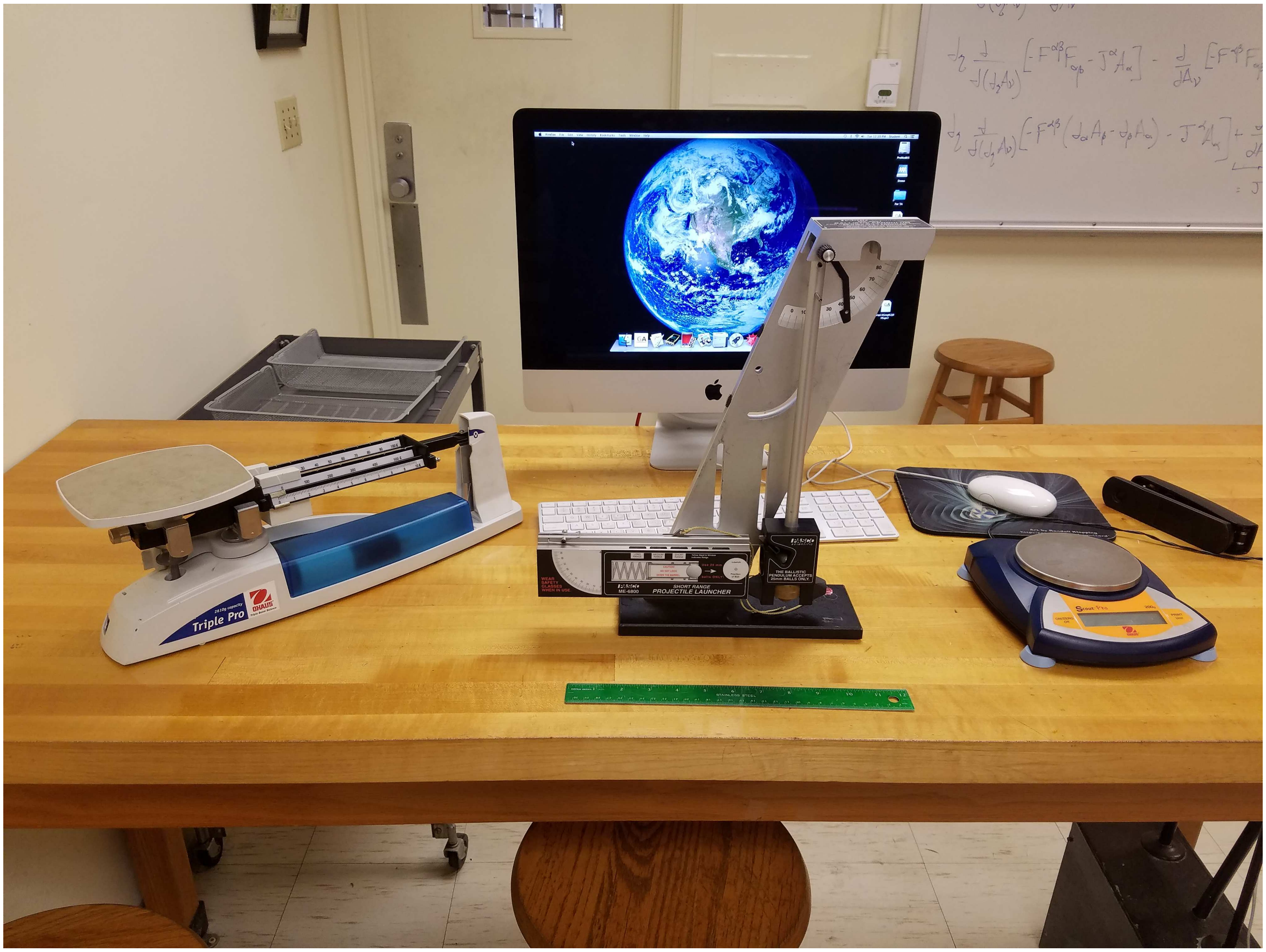

Figure 7.1: Ballistic Pendulum and Equipment

EQUIPMENT

Ballistic Pendulum
30 cm Ruler
Digital Balance
Triple-Beam Balance

Advance Reading

Text: Conservation of Energy, Conservation of Linear Momentum, Mechanical Energy, Kinetic Energy, Gravitational Potential Energy, Elastic Potential Energy, Elastic and Inelastic Collisions.

Objective

To determine the velocity of a ball as it leaves the ballistic pendulum using conservation of linear momentum and conservation of energy considerations; to perform error analysis.

Theory

Energy is always conserved. There are different forms of energy, and one form of energy may be transformed into another form of energy. **Mechanical energy**, a form of energy being investigated today, is not always conserved.

The mechanical energies being investigated today are kinetic energy, KE, and gravitational potential energy, PE_{grav}:

$$KE = \frac{1}{2}mv^2 \tag{7.1}$$

where a mass, m, has energy due to its speed, v, and

$$PE_{grav} = mgh \tag{7.2}$$

where a mass m has energy as a result of its position (height, h), and g is acceleration due to gravity.

Linear momentum, $\vec{\mathbf{p}}$, is always conserved in an *isolated system*. An isolated system is a system in which *all forces* acting on the system are considered.

Linear momentum p is given by:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} \tag{7.3}$$

where a mass, m, has a velocity, v. There are three distinct categories of collisions: elastic, inelastic, and completely inelastic.

Elastic collisions result in conservation of both linear momentum and mechanical energy. Billiard balls are often used as examples when discussing elastic collisions.

Inelastic collisions result in deformation of one or more of the objects involved in the collision. Although linear momentum is conserved, mechanical energy is not. Car wrecks are examples of inelastic collisions.

Completely inelastic collisions refer to collisions that result in objects becoming attached to each other after the collision (*i.e.*, stuck together). The objects thus move, after collision, with the same velocity. These collisions, like inelastic collisions, conserve linear momentum but not mechanical energy.

This experiment investigates completely inelastic collisions to determine the initial velocity of the ball. When the ballistic pendulum is fired, the ball is caught and held by the catcher; the two objects, ball and catcher, move together as one object after collision. We assume that the linear momentum of the system before and after the collision is conserved, and that no energy is lost during the ball's flight.

First Process:

Conservation of linear momentum between state 1 and state 2 (just before and just after a collision) is:

$$\vec{\mathbf{p}}_\mathbf{1} = m\vec{\mathbf{v}}_\mathbf{1} = \vec{\mathbf{p}}_\mathbf{2} = m\vec{\mathbf{V}}_\mathbf{2} \tag{7.4}$$

Relevant to this experiment, we consider a collision between a ball of mass m and a catcher of mass M. Before the collision, state 1, the velocity of the catcher is 0.0 m/s. *Just after* the collision, state 2, the velocity of the ball and the catcher are equal.

$$\vec{\mathbf{p}}_\mathbf{1} = m\vec{\mathbf{v}}_\mathbf{1} = \vec{\mathbf{p}}_\mathbf{2} = (m+M)\vec{\mathbf{V}}_\mathbf{2} \tag{7.5}$$

Second Process:

While mechanical energy is not conserved *during* an inelastic collision. However, *after* the collision, the ball-catcher system has KE due to its motion. We assume that mechanical energy is conserved (*i.e.*, ignore rotational energy and energy losses due to friction). KE is transformed into PE_{grav} as the pendulum arm swings up to a height h.

The initial mechanical energy is all KE, as the pendulum arm is at the lowest possible position. The final energy is all PE_{grav} when the pendulum rises and stops, state 3. Therefore, conservation of mechanical energy is:

$$KE = \frac{1}{2}(m+M)V_2^2 = PE_{grav} = (m+M)gh \tag{7.6}$$

When a pendulum rotates $\theta°$, the center of mass, cm, rises an amount $h = h_f - h_i$. Refer to Fig. 7.2.

By measuring the change in height, h, of the center of mass of the pendulum arm, PE_{grav} can be determined. A mark is scribed on the pendulum arm, just above the catcher. Once h is determined, one can calculate V_2 using Eq. 7.6, which, in turn, allows calculation of v_1. *Note* the use of capital V for V_2; this is to distinguish it from the notation used for the initial velocity of the ball when fired from detent 2, v_2.

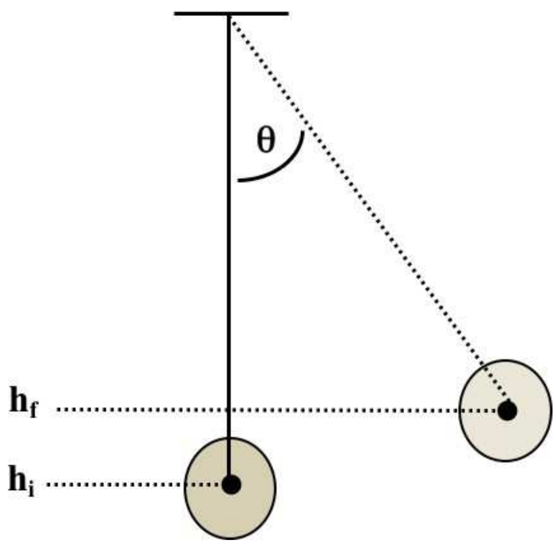

Figure 7.2: The change in height of the center of mass is $h \equiv h_f - h_i$

Name: ____

1. State the *Law of Conservation of Linear Momentum.* (10 pts)

2. State the *Principle of Conservation of Mechanical Energy.* (10 pts)

3. Solve the following equations (2 equations with 2 unknowns) for x in terms of: m, g, h. Refer to *Appendix D: Math Review* if necessary. (10 pts)

   $6x = 9y$

   $5y^2 = mgh$

4. Solve the following equations (2 equations with 2 unknowns) for x in terms of: m, M, g, h. (20 pts)

   $mx = (m + M)y$

   $\frac{1}{2}(m + M)y^2 = (m + M)gh$

(continued on next page)

5. Solve Eq. 7.5 and Eq. 7.6 (2 equations with 2 unknowns) for v_1 in terms of: m, M, g, h. (20 pts)

   $mv_1 = (m + M)V_2$ (Eq. 7.5)

   $\frac{1}{2}(m + M)V_2^2 = (m + M)gh$ (Eq. 7.6)

6. You shoot a ball, $m = 50.0$ g, into a catcher, $M = 200.0$ g; the center of mass rises 15.0 cm. Calculate v_1. Refer to your answer for *Question 5.* (20 pts)

7. You will fire the spring gun 3 times from the first detent and measure the change in height of the (pendulum + ball) for each shot. Write the equation for the change in height of the first shot. (10 pts)

PROCEDURE

1. Measure the mass of the ball (m) and catcher arm (M). [Unscrew the knurled nut to remove the arm; screw it back when finished.]
2. Measure the radius to the center of mass of the ball/pendulum system. The center of mass is the top of the opening of the ball catcher.
3. Fire the ball into the catcher and measure the change in height of the ball/pendulum system. $\Delta h = r - r\cos\Theta$
4. Record Δh for 3 trials at the first detent (short range). Calculate the average Δh for this detent. Record it in the table provided.
5. Conservation of energy dictates that the total energy remains constant throughout the motion of the pendulum [the force of friction is ignored]. Use this knowledge of energy conservation to determine the velocity of the ball + pendulum just after collision.
6. Write an equation relating the value of the conserved quantity (or quantities) before and after the collision. Calculate the initial velocity of the ball before collision, $\vec{\mathbf{v}}_{\mathbf{i}}$.
7. Repeat *Step 3* through *Step 4* for detent 2 (medium range) to determine $\vec{\mathbf{v}}_{\mathbf{2}}$.
8. Repeat *Step 3* through *Step 4* for detent 3 (long range) to determine $\vec{\mathbf{v}}_{\mathbf{3}}$.

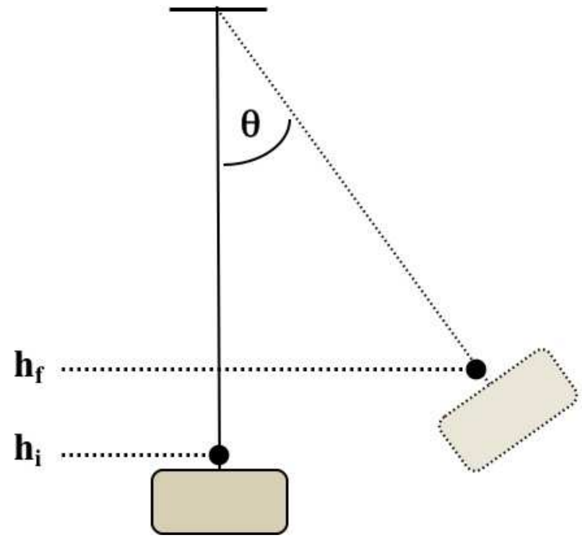

Figure 7.3: The *cm* of the *pendulum + ball* is just above the cylinder of the pendulum.

QUESTIONS

1. For each detent, calculate the momentum of the ball just after it is fired. (Assume an isolated system.)
2. Assume that the ballistic pendulum was not held firmly to the table when it was fired. What effect would this have on v_1?
3. Write the algebraic equation for mechanical energy *just before* the collision between the ball and the pendulum arm.
4. Write the algebraic equation for mechanical energy *just after* the collision between the ball and the pendulum arm.
5. For each detent:
   - Calculate the mechanical energy of the system *just before the collision.*
   - Calculate the mechanical energy of the system *just after the collision.*
   - What percent of the initial mechanical energy remains after the collision?
6. What happened to the "lost" mechanical energy?