Experiment 19: Exponentials and RC Circuits

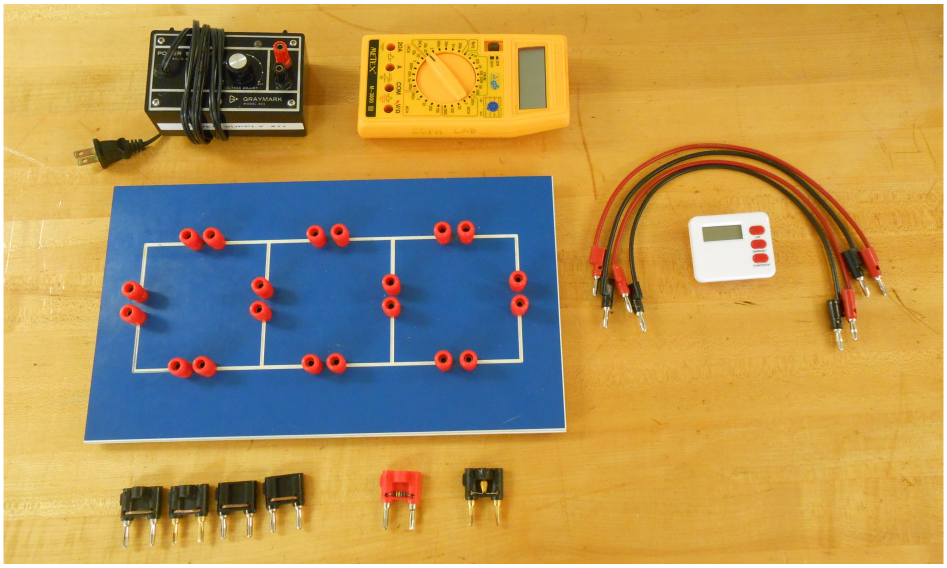

Figure 19.1: *Part 1*: RC Circuit

EQUIPMENT

Universal Circuit Board
(1) 680 kΩ Resistor
(4) Jumpers
(1) 47 μF Capacitor
(4) Wire Leads
Digital Multi-Meter (DMM)
Power Supply
Stopwatch

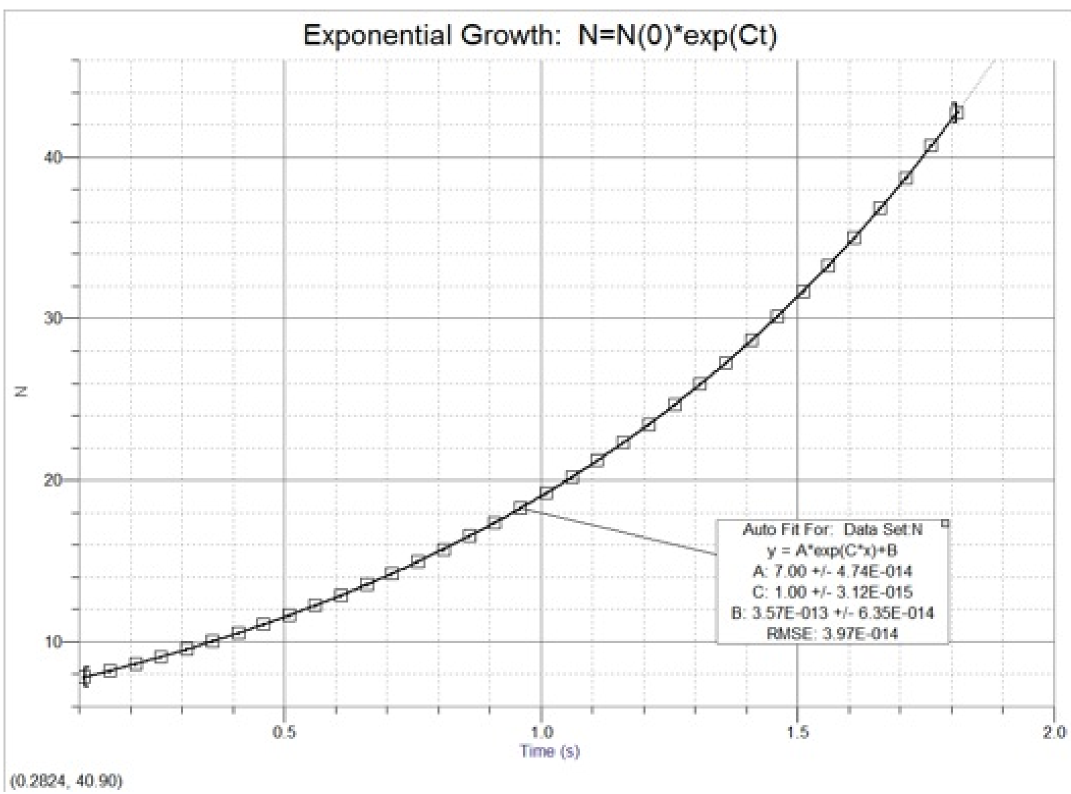

Figure 19.2: Exponential growth general equation is $N = N_0 e^{\lambda t}$
Exponential decay general equation is $N = N_0 e^{-\lambda t}$

Advance Reading (Exponentials)

Text: Exponential decay, RC circuit

Lab Manual: Appendix C

Objective

The objective of *Part 1* is to investigate an exponential curve by analysis of an RC circuit. Our RC circuit will be a resistor R and a capacitor C connected in parallel.

Theory

When the rate of change of a quantity is proportional to the initial quantity, there is an exponential relationship.

An exponential equation is one in which e is raised to a power. e is the irrational number 2.7182818....

An exponential equation has the form:

$$A(t) = A_0 e^{Bt} + C \tag{19.1}$$

where A is the amount or number after a time t and A_0 is the initial amount or number.

Exponential rates are found everywhere in nature. Some examples include exponential growth (*e.g.*, population of Earth) and exponential decay (*e.g.*, decay of radioactive elements), as well as heating and cooling rates.

Analysis of exponential curves often involves the *doubling-time* or *half-life* ($T_{1/2}$). Doubling-time is the time required for an initial amount to double in quantity (what is t when $A = 2A_0$?). The half-life is the time required for 1/2 of the initial amount to be gone (what is t when $A = \frac{1}{2}A_0$?).

Analyzing these curves is often simple. For doubling-time or half-life, begin with the appropriate value on the *y-axis*. Then draw a horizontal line that intersects the plot. Next, drop a perpendicular line to the *x-axis*. Read the value of the time directly from the *x-axis*.

The data for our exponential curve will be obtained by measuring the potential difference across the resistor of an RC circuit as a function of time.

A capacitor is connected in parallel with a resistor, then charged to some initial voltage. When the power supply is disconnected, the potential difference across the capacitor will decrease exponentially. The voltage, V, across the capacitor as it discharges is given by:

$$V_t = V_0 e^{-t/RC} \tag{19.2}$$

where V_0 is the initial potential difference across the capacitor at time t_0, R is resistance, and C is capacitance.

Consider Eq. 19.2. When $t = RC$,

$$V_t = V_0 e^{-1} = V_0 \frac{1}{e} \tag{19.3}$$

We define τ (Greek letter, tau) to be the time it takes for the voltage to drop to $(1/e) \cdot (V_0)$ its original value (about 37%). This is the value when $\tau = RC$.

For our discharging capacitor:

A graph of *Voltage vs. Time* is an exponential decay curve. Analysis of this curve provides the time constant by locating the point at which V has dropped to $1/e$ its original value of V_0.