Experiment 7: Conservation of Energy and Linear Momentum

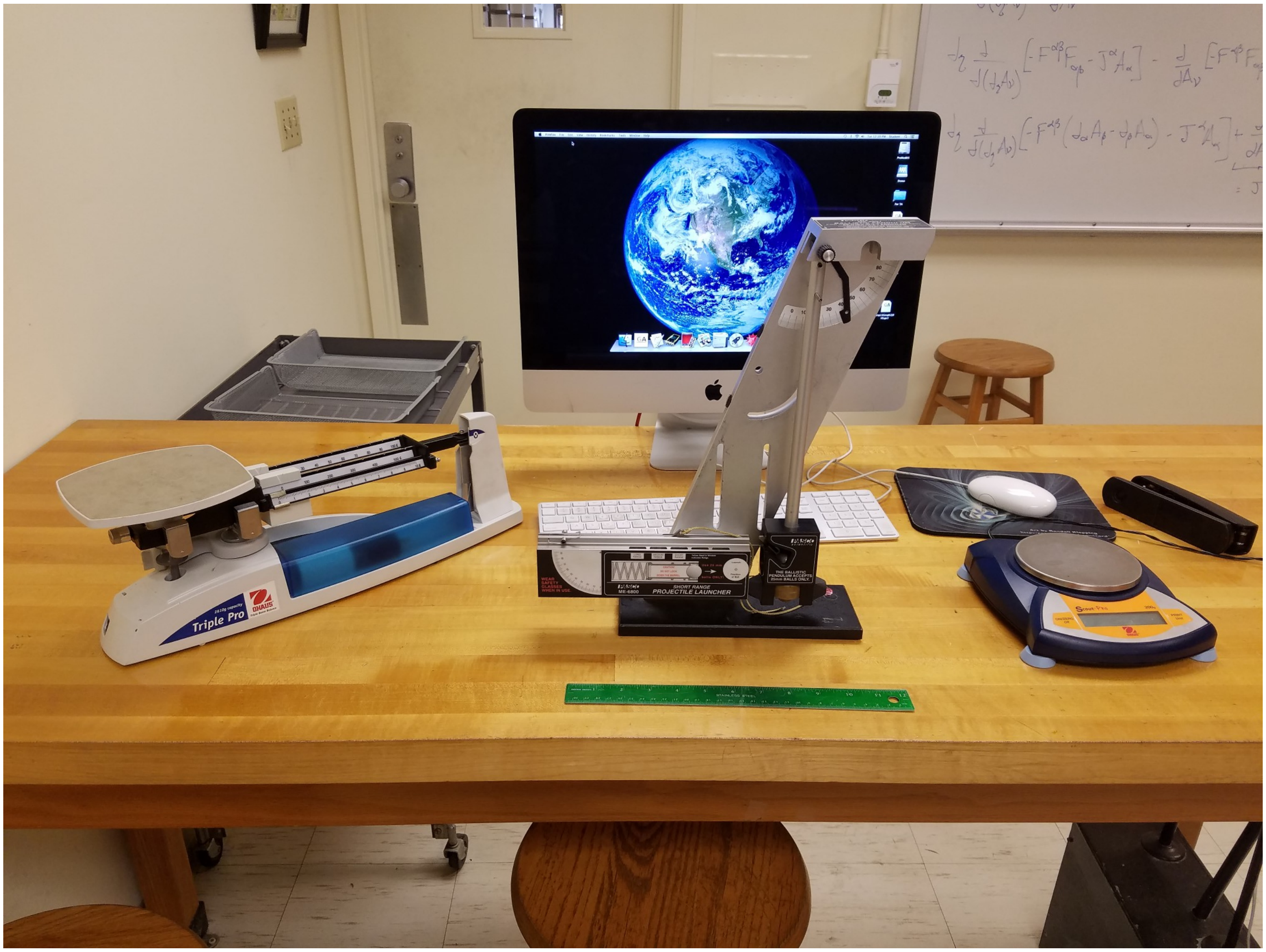

Figure 7.1: Ballistic Pendulum and Equipment

EQUIPMENT

Ballistic Pendulum
30 cm Ruler
Digital Balance
Triple-Beam Balance

Advance Reading

Text: Conservation of Energy, Conservation of Linear Momentum, Mechanical Energy, Kinetic Energy, Gravitational Potential Energy, Elastic Potential Energy, Elastic and Inelastic Collisions.

Objective

To determine the velocity of a ball as it leaves the ballistic pendulum using conservation of linear momentum and conservation of energy considerations; to perform error analysis.

Theory

Energy is always conserved. There are different forms of energy, and one form of energy may be transformed into another form of energy. **Mechanical energy**, a form of energy being investigated today, is not always conserved.

The mechanical energies being investigated today are kinetic energy, KE, and gravitational potential energy, PE_{grav}:

$$KE = \frac{1}{2}mv^2 \tag{7.1}$$

where a mass, m, has energy due to its speed, v, and

$$PE_{grav} = mgh \tag{7.2}$$

where a mass m has energy as a result of its position (height, h), and g is acceleration due to gravity.

Linear momentum, $\vec{\mathbf{p}}$, is always conserved in an *isolated system*. An isolated system is a system in which *all forces* acting on the system are considered.

Linear momentum p is given by:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} \tag{7.3}$$

where a mass, m, has a velocity, v. There are three distinct categories of collisions: elastic, inelastic, and completely inelastic.

Elastic collisions result in conservation of both linear momentum and mechanical energy. Billiard balls are often used as examples when discussing elastic collisions.

Inelastic collisions result in deformation of one or more of the objects involved in the collision. Although linear momentum is conserved, mechanical energy is not. Car wrecks are examples of inelastic collisions.

Completely inelastic collisions refer to collisions that result in objects becoming attached to each other after the collision (*i.e.*, stuck together). The objects thus move, after collision, with the same velocity. These collisions, like inelastic collisions, conserve linear momentum but not mechanical energy.

This experiment investigates completely inelastic collisions to determine the initial velocity of the ball. When the ballistic pendulum is fired, the ball is caught and held by the catcher; the two objects, ball and catcher, move together as one object after collision. We assume that the linear momentum of the system before and after the collision is conserved, and that no energy is lost during the ball's flight.

First Process:

Conservation of linear momentum between state 1 and state 2 (just before and just after a collision) is:

$$\vec{\mathbf{p}}_\mathbf{1} = m\vec{\mathbf{v}}_\mathbf{1} = \vec{\mathbf{p}}_\mathbf{2} = m\vec{\mathbf{V}}_\mathbf{2} \tag{7.4}$$

Relevant to this experiment, we consider a collision between a ball of mass m and a catcher of mass M. Before the collision, state 1, the velocity of the catcher is 0.0 m/s. *Just after* the collision, state 2, the velocity of the ball and the catcher are equal.

$$\vec{\mathbf{p}}_\mathbf{1} = m\vec{\mathbf{v}}_\mathbf{1} = \vec{\mathbf{p}}_\mathbf{2} = (m+M)\vec{\mathbf{V}}_\mathbf{2} \tag{7.5}$$

Second Process:

While mechanical energy is not conserved *during* an inelastic collision. However, *after* the collision, the ball-catcher system has KE due to its motion. We assume that mechanical energy is conserved (*i.e.*, ignore rotational energy and energy losses due to friction). KE is transformed into PE_{grav} as the pendulum arm swings up to a height h.

The initial mechanical energy is all KE, as the pendulum arm is at the lowest possible position. The final energy is all PE_{grav} when the pendulum rises and stops, state 3. Therefore, conservation of mechanical energy is:

$$KE = \frac{1}{2}(m+M)V_2^2 = PE_{grav} = (m+M)gh \tag{7.6}$$

When a pendulum rotates $\theta°$, the center of mass, cm, rises an amount $h = h_f - h_i$. Refer to Fig. 7.2.

By measuring the change in height, h, of the center of mass of the pendulum arm, PE_{grav} can be determined. A mark is scribed on the pendulum arm, just above the catcher. Once h is determined, one can calculate V_2 using Eq. 7.6, which, in turn, allows calculation of v_1. *Note* the use of capital V for V_2; this is to distinguish it from the notation used for the initial velocity of the ball when fired from detent 2, v_2.

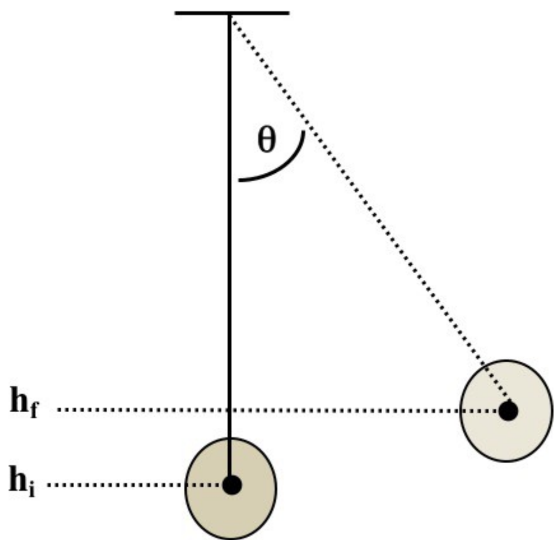

Figure 7.2: The change in height of the center of mass is $h \equiv h_f - h_i$