

Fig. 3-1 Force Table

**Equipment**

Force Table  
Ring and string  
4 Pulleys  
4 Weight Hangers  
Masses  
Protractor  
30-cm Ruler  
Two-Force Card  
Three-Force Card  
Level

# Experiment 3

## Vector Addition

**Advance Reading:** University Physics Vol. 1 by OpenStax, Chapter 2 and Chapter 5

**Objective:** The objective of this lab is to study vector addition by three different methods (component method, graphical method and using a force table.)

**Theory:** Vectors are quantities that have both magnitude and direction and follow specific rules of combination. Two of these rules are the *commutative and associative laws of addition*. The graphical method will be used to verify this rule.

In this lab, a force table will be used to check the graphical and component methods of vector addition. The table is a circular steel table that has the angles  $0^\circ$  to  $360^\circ$  inscribed on the edge. (See Figure 3-1.) To use the force table, pulleys are placed at the angles specified by the force cards, with the strings attached to the center ring running over the pulleys. Masses are placed on weight hangers attached to the end of the strings to provide the force needed. By adding the vectors, the resultant vector is found. To balance the force table, however, a force that is equal in magnitude and opposite in direction must counter-balance the resultant. This force is the **equilibrant**. For example, if a 10 N force at  $0^\circ$  and a 10 N force at  $90^\circ$  are added the resultant vector has a magnitude of 14.7 N at  $45^\circ$ . The equilibrant has the same magnitude, but the direction is  $180^\circ + 45^\circ = 225^\circ$ .

### Procedure:

#### Part 1a: Graphical Method- Commutative property of vector addition using two vectors

1. There should be a two-force card and a three-force card on your table. Choose the **two-force card**. Draw directed line segments for each force. Each should begin at the origin of an x-y axis that has been drawn at the center of your graph paper. Let 1 cm equal 1 N. Label one vector  $\mathbf{F}_1$  and the other  $\mathbf{F}_2$ .
2. One lab partner should redraw the vector  $\mathbf{F}_2$  at the tip of vector  $\mathbf{F}_1$  and then draw a line connecting the origin at the tip of vector  $\mathbf{F}_2$ . This line is vector  $\mathbf{F}_1 + \mathbf{F}_2$ . See Fig. 3-2.

The second lab partner should draw the vector  $\mathbf{F}_1$  at the tip of vector  $\mathbf{F}_2$  and then draw line connecting the origin at the tip of vector  $\mathbf{F}_1$ . This line is also the vector  $\mathbf{F}_2 + \mathbf{F}_1$ .

*Please note in Fig. 3-2, the bottom triangle represents  $\mathbf{F}_1 + \mathbf{F}_2$ , while the top triangle represents  $\mathbf{F}_2 + \mathbf{F}_1$ . Thus it can be seen that vectors commute when added since the resultant vector is the same (i.e.,  $\mathbf{F}_1 + \mathbf{F}_2 = \mathbf{C} = \mathbf{F}_2 + \mathbf{F}_1$ ).*

3. Measure the angle and the magnitude of the resultant vector with the protractor and the ruler. Write down you lab partner's answer.

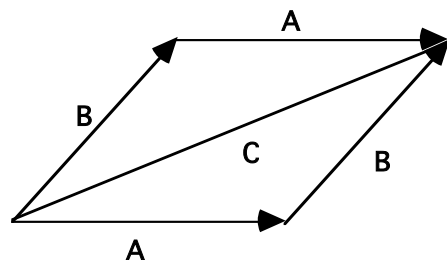


Fig. 3-2  $\mathbf{A} + \mathbf{B} = \mathbf{C} = \mathbf{B} + \mathbf{A}$

## Part 1b: Graphical Method 2 – Associative property of vector addition using three vectors

4. Choose the three-force card and draw *any one of the three forces* from the origin. This is your starting vector. One lab partner should start with vector  $\mathbf{F}_1$  (and add  $\mathbf{F}_2$ , then  $\mathbf{F}_3$ ). The other lab partner should start with vector  $\mathbf{F}_2$  and add  $\mathbf{F}_3$  then  $\mathbf{F}_1$ .
5. Select either of the two remaining forces then place it (the 2nd second force chosen) at the tip of the first. Continue this procedure until all forces are drawn. The resultant is drawn from the origin to the tip of the last force drawn. Measure the magnitude and direction of the resultant. Both lab partners should have a different polygon, but the same resultant. Write down your lab partner's result.

## PART 2: Analytical Method

If the direction of a vector is measured *from* the positive x-axis in a counter-clockwise direction, (standard procedure), then the following is true:

$$F_x = F \cos \theta \quad \{\text{magnitude of the x-component}\}$$

$$F_y = F \sin \theta \quad \{\text{magnitude of the y-component}\}$$

$$\theta = \tan^{-1}(F_y/F_x) \quad \{\text{direction of the vector}\}$$

When you add vectors, they yield a **resultant**,  $\mathbf{R}$ . To add vectors mathematically, you need to first determine the x and y-components of each vector.  $F_x$  is the sum of the x-components and  $F_y$  is the sum of the y-components. Thus

$$R = \sqrt{F_x^2 + F_y^2} \quad \{\text{magnitude of } \mathbf{R}\}$$

$$\theta = \tan^{-1}(F_y/F_x) \quad \{\text{direction of } \mathbf{R}\}$$

6. Calculate the magnitude and direction of  $\mathbf{R}$  for your forces. *Note:* Verify that the angle  $\theta$  is in the proper quadrant based upon your  $F_x$  and  $F_y$ . Your calculator will give you only one of two possible angles (the correct angle or the correct angle plus 180 degrees).

## Part 3: Force Table

7. If values from parts one and two agree, use the force table to verify these answers. *If necessary, level the force table using the three screws attached to the base of the table and the carpenter's level.* Place pulleys at the positions specified by the force card; add masses to provide the forces. Use 100 grams to represent each newton of force. If the values obtained for the resultant are correct, then the equilibrant will balance the system and the ring will be positioned in the middle of the force table. Add the equilibrant force to the proper position.
8. To determine the uncertainty in the magnitude and direction of the equilibrant:  
 $\delta m$  - add mass to the equilibrant until the ring shifts slightly (i.e., some but not too much).  
 $\delta \theta$  - adjust the position ( $\theta$ ) until the ring shifts (again, some but not too much).

## Post Lab Questions: (You must show all work to receive full credit.)

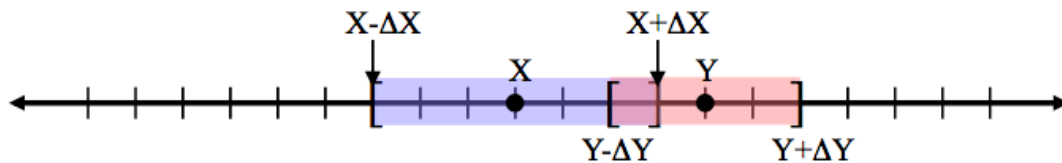
1. In this experiment you determined the length of a vector by three different methods (which should have three different uncertainties.) Were all of your answers the same when uncertainty is

considered (for both 2 and 3 vectors)? **The resulting answer to this question is of fundamental importance and should be discussed in Results.**

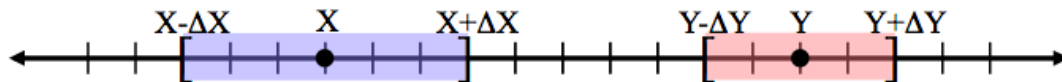
This question is answered by looking at **whether or not all three answers overlap when certainty is considered.** See the text below (which was taken from the website of Dr. David Brooks of Florida International University.

<http://www2.fiu.edu/~dbrookes/ExperimentalUncertaintiesCalculus.pdf>).

Thus, to make judgment about two values  $X$  and  $Y$ , you have to find the ranges where these values lie. If the ranges  $X \pm \Delta X$  and  $Y \pm \Delta Y$  overlap, you can claim that the values  $X$  and  $Y$  agree within your experimental uncertainty.



If the two values  $X$  and  $Y$  do not agree to within their experimental uncertainties, we say they are different.



Generate figures similar to the figure above to answer this lab question. Show all work.

2. The uncertainty in length of the vectors that were used for the analytical method (i.e.,  $0.0002 \text{ N} \Rightarrow 0.0002 \text{ cm}$  since it was assumed that  $1 \text{ cm} = 1 \text{ N}$ ) was the uncertainty of a length measured with a Kanon dial type **analog** caliper. What is the readability of the caliper in both cm and inches?
3. Instead of **adding** the given two vectors in this experiment you should **subtract** them by both the analytic (i.e., component) method and graphically. In other words, instead of  $\mathbf{F}_1 + \mathbf{F}_2$  or  $\mathbf{F}_2 + \mathbf{F}_1$  you are to determine either  $\mathbf{F}_1 - \mathbf{F}_2$  or  $\mathbf{F}_1 - \mathbf{F}_1$ . Show all work. Graph paper can be found online.

4. You and your lab partner added vectors in two ways. One lab partner should have added  $\mathbf{F}_1 + \mathbf{F}_2$  and the other lab partner  $\mathbf{F}_2 + \mathbf{F}_1$ . Did your results 'verify' the commutative law of addition for vectors. Explain your answer. Be sure and include uncertainties.