

# Experiment 10

## Moment of Inertia

### Advanced Reading

University Physics I by OpenStax, Ch 10, section 10.4-10.7.

### Equipment

- Beck's Inertia Thing (rotational apparatus)
- Vernier caliper
- masses
- meter stick
- stopwatch (cell phones are ok to use)

### Objective

The objective of this experiment is to dynamically measure the moment of inertia of a rotating system and to compare this to a predicted value.

### Theory

The moment of inertia can be viewed as the rotational analog of *mass*. Torque and angular acceleration are the rotational analogs of *force* and *acceleration*, respectively. Thus, in rotational dynamics, Newton's second Law ( $F=ma$ ) becomes  $\tau = I\alpha$ , where  $\tau$  is the (net) applied torque,  $I$  is the moment of inertia of the body and  $\alpha$  is the angular acceleration.

An object that experiences constant angular acceleration must have a constant torque applied to it. By applying a known torque to a rigid body, measuring the angular acceleration, and using the relationship  $\tau = I\alpha$ , the moment of inertia  $I$  can be found.

In this experiment, a torque is applied to the rotational apparatus by a string that is wrapped around the axle of the apparatus (Fig. 9-1).



Figure 9-1

The tension  $T$  is supplied by a hanging weight  $mg$ . The tension is found by applying Newton's second law (Fig. 9-2).

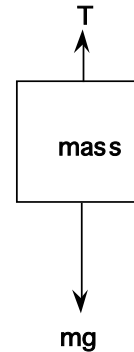


Figure 9-2 Forces on the hanging mass.

If we take upward direction as positive, and apply Newton's second law, we have

$$\Sigma F = T - mg = -ma ,$$

so the tension is

$$T = m(g - a)$$

The rotational apparatus has an original moment of inertia  $I_0$  with no additional masses added. When additional masses are added, it has a new moment of inertia  $I_{\text{new}}$ . The relationship between  $I_0$  and  $I_{\text{new}}$  is given by  $I_{\text{new}} = I_0 + MR^2$  where  $M$  is the total added mass and  $R$  is the distance of this mass from the center of the wheel (i.e., from the axis of rotation). *Please note that it is assumed that the added masses are point masses. We will explore whether this is an appropriate assumption in the questions.*

### Procedure

#### Part 1. Moment of Inertia of apparatus without additional weights

1. Using the vernier caliper, measure the diameter of the axle around which the string wraps. Calculate the radius  $r_{\text{axle}}$ . Make sure that no additional masses are added to the apparatus (Fig. 9-1).
2. Wrap the string around the axle and place enough weight on the string to cause the apparatus to rotate at a constant speed (or very slowly). The angular acceleration should be nearly zero. When this is the case, the sum of the torques on the body

must be (nearly) zero. The amount of mass needed is on the order of a few grams. From this data, calculate the frictional torque which is given by

$$\tau_{friction} = r_{axle} F_{friction} = r_{axle} m_{friction} g.$$

3. REMOVE FRICTION MASS. Place a 50 gram mass hanger on string. Measure the distance from the bottom of the weight hanger to the floor. Release the weight hanger, being sure not to impart an initial velocity to the wheel of the rotational apparatus.

4. Use the stopwatch to time the fall. Perform a total of five trials and calculate an average distance and an average time. From this information, calculate the acceleration of the mass using  $distance = 1/2 at^2$ . Calculate the angular acceleration  $\alpha = a/r$ , where  $r$  is the radius of the axle that the string is wrapped around.

5. Next, calculate the tension of the string. (See the Theory section.)

6. The applied torque on the spinning wheel is provided by the tension of the string. Use the value of the tension to calculate this torque. Next, **calculate the net torque**, which is the **applied torque minus the frictional torque**.

7. Add 50 grams to hanger for a total of 100 grams. Repeat steps 3-6.

8. **Plot the net torque vs. angular acceleration. Be sure to enter the origin as a data point.** Determine the moment of inertia  $I_0$ , which is the slope of the best-fit line.

## Part II Additional masses

9. Measure the distance from the center of the inertia wheel to the center of the outer set of tapped-holes. Do this for all three arms and average the distance. (*They should all be nearly the same.*)

11. Add the total mass of the three brass masses. The mass of each one is written on the side and/or top. Attach the masses to the apparatus. Calculate the new moment of inertia,  $I_{new}$ , with these additional masses located at a distance  $R$  from the axis of rotation. **Measure the diameter of the masses. This information will be needed to answer question 3.**

12. Repeat the steps 2 through 8 for this new moment of inertia. Plot the data and determine

moment of inertia  $I_{new}$  from the slope. Calculate the percent difference between the experimental value and the calculated value.

## Questions/Conclusions

1. What is the maximum kinetic energy that the inertia device (wheel) used is given by the hanging mass (just before the mass reaches the floor). Compare this value to the gravitational energy that the hanging mass has just before you release it. **Choose only one of your 4 runs and say which you used.** Show all work.

2. If the torque applied to a rigid body is doubled, what happens to the moment of inertia of the body? What happens to the angular acceleration? Explain your answers.

3. In the theoretical determination of the moment of inertia  $I_{new}$  with the additional masses, it was assumed that the masses are points.

Using the parallel-axis theorem, calculate the moment of inertia such that the diameter of the masses is taken into account. Determine the percentage difference between this and the previous value. Is it a good approximation to assume that the masses are points in this particular case?

4. What is the moment of inertia of a drum which is rotated about the vertical axis which runs through the center of the drum. Assume drum is a hoop. You should ignore the top and bottom pieces of drum and assume drum is a hoop. **See table below for which drum you should use. Table from website <https://www.skolnik.com/steel-drum-measurements>**

## Steel Drum Measurements

CAPACITY Gallons   Liters	MEASUREMENTS			MAX WEIGHT & GAUGE	
	Inside	US	Metric	US	Metric
sec 1 3   11	Diameter	14"	36cm	8 lbs.	4 kg
	Height	6"	15cm	20 gauge	0.9 mm
sec 2 5   19	Diameter	14"	36cm	10 lbs.	4.5 kg
	Height	10"	25cm	20 gauge	0.9 mm
sec 3 8   30	Diameter	14.00"	35 cm	12 lbs.	5 kg
	Height	14.00"	36 cm	20 gauge	0.90 mm
sec 4 10   38	Diameter	14.00"	35 cm	14 lbs.	6 kg
	Height	17.00"	43 cm	20 gauge	0.90 mm
sec 5 16   60	Diameter	14"	36cm	19 lbs.	9 kg
	Height	27"	70cm	20 gauge	0.9 mm
sec 6 20   76	Diameter	18.25"	46 cm	30 lbs.	14 kg
	Height	19.25	49 cm	18 gauge	1.20 mm
sec 8 30   114	Diameter	18.25"	46 cm	35 lbs.	16 kg
	Height	27.50"	70 cm	18 gauge	1.50 mm
sec 9 55   208	Diameter	22.50"	57 cm	60 lbs.	27 kg
	Height	33.00"	84 cm	16 gauge	1.50 mm
sec 10 85   322	Diameter	26.00"	66 cm	80 lbs.	35 kg
	Height	37.00"	92 cm	16 gauge	1.50 mm
sec 12 96   363	Diameter	26.00"	66 cm	85 lbs.	39 kg
	Height	41.00"	104 cm	16 gauge	1.50 mm
sec 13 110   416	Diameter	30.00"	76 cm	102 lbs.	46 kg
	Height	41.00"	104 cm	16 gauge	1.50 mm