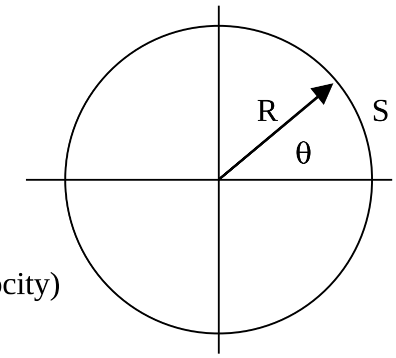
CHAPTER 10- ROTATION of a FIXED BODY ABOUT a FIXED AXIS\

ANGULAR DISPLACEMENT, VELOCITY, & ACCELERATION



In polar coordinates we have the fundamental relation
That the arc length inscribed by a radius R and angle θ
is just $S = R\,\theta$. (S ≡ angular displacement)

Now imagine the arrow tip sweeping counterclockwise
Such that S increasing at a constant rate
$V = dS/dt = R\, d\theta/dt = R\,\omega$. $\omega \equiv d\theta/dt$ (angular velocity)

$V = R\,\omega$

If the arrow tip is accelerating $a = d^2Vdt^2 = R\, d^2\theta / dt^2 = R\,\alpha$

$a = R\,\alpha$ ($\alpha \equiv$ angular acceleration)

θ, ω, α are measured in radians (rd), rd/s, rd/s^2 .respectively. These quantities are indedepent of R and only depend on the and θ. By multiplying the angular quantities θ, ω , α , by R , we get the linear quantities S, V, and a!

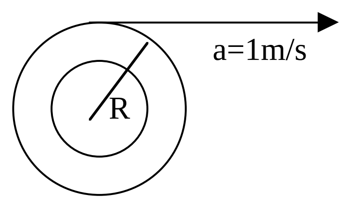
ANGULAR KINEMATICS EQUATION (constant acceleration)

The linear and angular kinematic equations are directly related.

LINEAR	ANGULAR	
$S(t) = So + Vo\,t + 1/2\,a\,t^2$	$\theta(t) = \theta o + \omega o\,t + 1/2\,\alpha\,t^2$	(1)
$V(t) = Vo + a\,t$	$\omega(t) = \omega o + \alpha\,t$	(2)
$a(t) = a$	$\alpha(t) = \alpha$	(3)
$V^2 = Vo^2 + 2\,a\,S$	$\omega(t)^2 = \omega o^2 + 2\,\alpha\,\Delta\theta$	(4)

Example

Consider a spool of rope radius R = 20cm, being unwound from rest with an acceleration of a=1 m/s. (a) After t=10s what is the angular velocity of the spool, (b) what length of thread has been unwound?



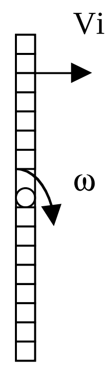
(a) Eq. (2) and $\alpha = a/R = (1/0.2) = 5$ rd/s^2

$\omega(10) = (\ 5$ rd/s^2) (10s) = <u>50 rd/s</u>

(b) Use (1) to find the angular displacement and multiply by R
To find Sà $S = R\,\theta$.
$\theta = 1/2\,\alpha\,t^2 = 1/2\,(5)\,(100) = 250$ rd and S = (1m)(250rd = <u>250m</u>

KINETIC ENERGY OF ROTATION

We can add (integrate) up all the kinetic energy from the parts (each mass = mi) of a rotating object:



$$K = \Sigma\ 1/2\ mi\ Vi^2 = 1/2\ \Sigma\ mi\ (Ri\ \omega)^2$$

We have used the fact that Vi = Ri ω

$$K = 1/2\ [\Sigma\ mi\ (Ri)^2\ ]\ \omega^2 = 1/2\ I\ \omega^2$$

$$\boxed{K = 1/2\ I\ \omega^2}$$ kinetic energy of rotation

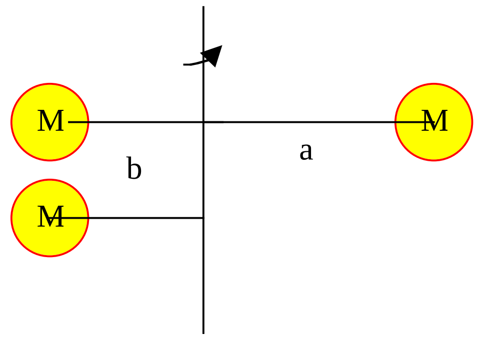
MOMENT OF INERTIA

We notice that each object will have a different moment of inertia *I* about the rotation point.

$$\boxed{I = \Sigma\ mi\ (Ri)^2 = \int r^2\, dm}$$

SUM FORMULA

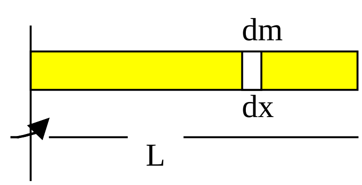
The moment of inertia of these sphere rotating about the z-axis is



$$Iz = Ma^2 + Mb^2 + Mb^2$$
$$= M\ (a^2 + 2b^2)$$

INTEGRAL FORMULA

$$Iz = \int r^2\, dm = \int_0^L x^2\, dm$$



dm = mass of ith slice dm = (Mass / Length) dx

dm = ρ dx

$$Iz = \int_0^L x^2\, \rho\, dx = (M/L)\ L^3 / 3 = \underline{M\ L^2 / 3}$$

(see pg 304 in text for others)

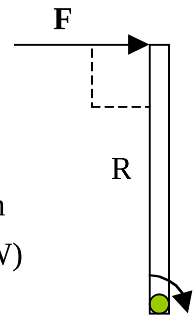
TORQUE and Newton's Laws

We can define a rotational force call torque $\tau = I\alpha$.

The sum of the torques on a rigid body must equal to the product of moment of iniertia and angular acceleration.

$$\boxed{\Sigma\tau = I\alpha}$$ Newton's Law for Rotation

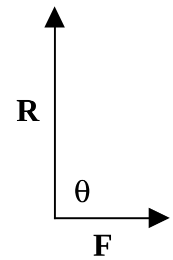
The torque below acts through a moment arm R. The force exerted thru the distance R gives the torque. We must always use the perpendicular component of the force with respect to the moment arm R.



$$\tau = R F_{perp}$$

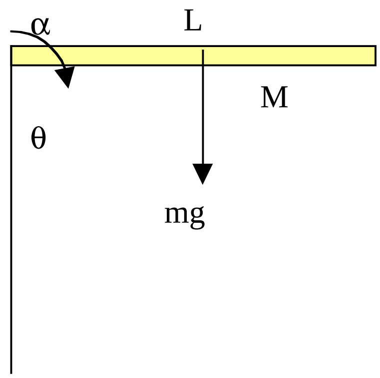
Torque τ is a vector quantity. τ's direction is along the axis of rotation. In this example τ is in to the page like turning a screw driver clockwise (CW) in to the page. A CCW rotation would give a τ out of the page.

The vector cross product is used to express the τ mathematically .



τ = **R** x **F** where the magnitude is $\tau = R F \sin\theta$.
We see that R points from pivot point to point of force contact.
And θ is the angle between **R** and **F** .

ROTATING ROD



(a) Find the angular accelertion of a pivoting rod of mass M and length L=1m.

The force F=mg acts through the center of mass of the rod.

$\Sigma\tau = mg (L/2) = I \alpha$ where $I = 1/3 ML^2$

$\alpha = mg (L/2I) = (3/2) g/L$

(b) What is the acceleration of the rod tip?
$a = L \alpha = 3/2 g$

This the tip of the rod is falling with acceleration faster than g!

(c) How long does it take the rod to fall to the vertical position ($\theta = 0^o$)?
$\pi/2 = 1/2 \alpha t^2$
$t = (\pi/\alpha)^{1/2} = \pi / (3/2g) =$ <u>0.46 s</u>

(d) How long will it take a coin placed on the rod to fall?
$t = (2L/g)^{1/2} = .45$ s

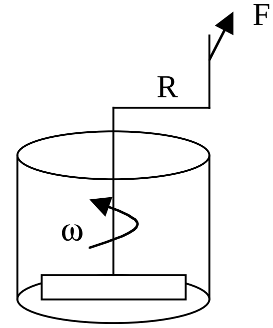
TORQUE and POWER

Power is defined as the work performed per unit time. $P = dW/dt$, where $W = \tau\ \Delta\theta$.
In angular terms this translates to $P = d(\tau\ \Delta\theta)/dt = \tau\ \omega$, measured in Watts.

$$\mathbf{P = \tau\ \omega}$$

We compare this to the formula for linear motion $P = F\ v$.

Example
JamesWatt applied a force F=10N to a handle of length R=50cm and turned the crank at a frequency of f = 1 turn a second how much power was he generating?



$P = \tau\ \omega = F\ R\ 2\pi\ f$ = (10N)(0.5m) (6.28) (1/s) = 31.4 W