**UNCERTAINTY AND ERROR**

Every measurement has some uncertainty. These uncertainties are called errors. “Error analysis is the study and evaluation of uncertainty in measurement.”¹ Measurements are usually made against some standard to compare the object or quantity being measured with some known value. For instance, if the length of a table is measured with a meter stick, the table is being compared to the meter stick, but the meter stick is also referenced to some standard. It is important to keep in mind that any “known” value given as a standard has an uncertainty associated with it. Any measurement you make has an uncertainty associated with it as well.

Error analysis is an interesting and complex subject. As an introduction to this topic, certain experiments will focus on particular types of analysis: uncertainty in measurement, statistical analysis, and propagation of error (uncertainty). The details follow.

**Significant Figures**

Use the rules for significant figures found in your text. All data should be recorded with the proper number of significant figures in your lab notebook as well as in your lab reports.

**Accuracy and Precision**

An important consideration in a laboratory situation is the accuracy and the precision of a measurement. Commonly, these two terms are used as synonyms, but they are quite different. The accuracy of a measurement is how close the measurement is to some “known” value (how small the percent error is; related to systematic and personal error). For instance, if an experiment is performed to measure the speed of light, and the experimental value is very close to the known (accepted) value, then it can be said that the value is accurate. On the other hand, the precision of an experiment is a measure of the reproducibility of an experiment (related to random error).

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When performing an experiment, one needs to keep in mind that a measurement that is precise is not necessarily accurate, and vice versa. For instance, a vernier caliper, an instrument used to measure lengths with a precision of 0.005 centimeter, could be used to measure the length of an object, but the vernier caliper has been damaged so that it reads one centimeter too short. The value is precise, but it is not accurate.

**EXPERIMENTAL ERRORS and COMPARISON**

Experimental errors are generally classified into three types: systematic, personal, and random.

**Systematic Error**

Systematic errors are such that measurements are pushed in one direction. Examples include a clock that runs slow, debris in a caliper that increases measurements, or a ruler with a rounded end that goes unnoticed, reducing measurements. To reduce this type of error all equipment should be inspected and calibrated before use.

**Personal Error**

Carelessness, personal bias, and technique are sources of personal error. Care should be taken when entering values in your calculator and during each step of the procedure. Personal bias might include an assumption that the first measurement taken is the “right” one. Attention to detail will reduce errors due to technique.

Parallax, the apparent change in position of a distant object due to the position of an observer, could introduce personal error. To see a marked example of parallax, close your right eye and hold a finger several inches from your face. Align your finger with a distant object. Now close your left eye and open your right eye. Notice that your finger appears to have jumped to a different position. To prevent errors due to parallax, always take readings from an eye-level, head-on, perspective.

**Random Error**

Random errors are unpredictable and unknown variations in experimental data. Given the randomness of the errors, we assume that if enough measurements are made, the low values will cancel
the high values. Although statistical analysis requires a large number of values, for our purposes we will make a minimum of six measurements for those experiments that focus on random error.

**Percent Error and Percent Difference**

We frequently compare experimental values with accepted values (percent error) and experimental values with other experimental values (percent difference).

Accepted values might be found in tables, calculated from equations, or determined experimentally (e.g., \( g = 9.80 \, \text{m/s}^2 \), \( c = 3.0 \times 10^8 \, \text{m/s} \)).

When we compare two or more experimental values, we use more than one method to determine that particular quantity.

When comparing values, use the following equations:

\[
\text{% error} = \frac{\text{accepted} - \text{experimental}}{\text{accepted}} \times 100
\]

\[
\text{% difference} = \frac{|\text{value}_1 - \text{value}_2|}{\frac{\text{value}_1 + \text{value}_2}{2}} \times 100 = \frac{|\text{value}_1 - \text{value}_2|}{\text{average}} \times 100
\]

**ERROR ANALYSIS**

**Random Error**

When analyzing random error, we will make a minimum of six measurements (N=6). From these measurements, we calculate an average value (mean value):

\[
\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_N}{N} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = \frac{1}{N} \sum_{i=1}^{N} x_i = \frac{1}{6} \sum_{i=1}^{6} x_i. \quad (1)
\]
To determine the uncertainty in this average, we first compute the deviation, $d_i$:

$$d_i = x_i - \bar{x}. \quad (2)$$

The average of $d_i$ will equal zero; therefore, we compute the standard deviation, $\sigma$.

Analysis shows that approximately 68% of the measurements made will fall within one standard deviation, while approximately 95% of the measurements made will fall within two standard deviations.

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (d_i)^2} = \sqrt{\frac{1}{5} \sum_{i=1}^{6} (d_i)^2} = \sqrt{\frac{1}{5} \sum_{i=1}^{6} (x_i - \bar{x})^2} \quad (3)$$

or

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + (x_3 - \bar{x})^2 + (x_4 - \bar{x})^2 + (x_5 - \bar{x})^2 + (x_6 - \bar{x})^2}{5}}. \quad (4)$$

**Measurement**

In this lab the uncertainty, $\delta$, of a measurement will be 1/2 of the smallest division of a measuring device. If an object is measured with a 30-cm ruler and the smallest division is one millimeter, the uncertainty of the measurement is 0.5 mm (0.05 cm). For example, an object is measured to be $(23.25 \pm 0.05)$ cm. The “5” in 23.25 is estimated. The measurement, $(23.25 \pm 0.05)$ cm, means that the true measurement is most likely between 23.20 cm and 23.30 cm.

**Propagation of Error**

Once the uncertainties for your measurements are known, they can be propagated in all mathematical manipulations that use the quantities you measured. This allows a reader to know the precision of your work. When we propagate error, the following rules will apply, depending on how you use your data.
Once the uncertainties in a set of data are known, these uncertainties must be propagated in all mathematical manipulations of the data. If $u(x,y,z)$ is some known function of the measured values, then to calculate $u$ and its uncertainty $\sigma_u$, we first consider the differential

$$\text{du} = \frac{\partial u}{\partial x} \text{dx} + \frac{\partial u}{\partial y} \text{dy} + \frac{\partial u}{\partial z} \text{dz}. \quad (5)$$

This relationship gives the change in $u$ due to small changes in $x$, $y$, and $z$. The symbol $\frac{\partial u}{\partial x}$ is the partial derivative of the function $u(x,y,z)$, which is found by taking the derivative of $u$ with respect to $x$ while treating the variables $y$ and $z$ as constants. For example, the partial derivatives for the function $u(x,y,z) = 6yx^2 - 5zy^3$ are

$$\frac{\partial u}{\partial x} = 12yx$$
$$\frac{\partial u}{\partial y} = 6x^2 - 15zy^2$$
$$\frac{\partial u}{\partial z} = -5y^3.$$

Due to the tendency of two or more errors to cancel, Eq. (5) is an overestimate of the uncertainty. The correct relationship for the uncertainty $\sigma_u$ of the function $u(x,y,z)$ is

$$\sigma_u = \sqrt{\left(\frac{\partial u}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial u}{\partial y}\right)^2 \sigma_y^2 + \left(\frac{\partial u}{\partial z}\right)^2 \sigma_z^2}, \quad (6)$$

where $\sigma_x$, $\sigma_y$, and $\sigma_z$ are the uncertainties in the measurements of $x$, $y$, and $z$, respectively.

Let us now consider special cases of Eq. (6). For addition and subtraction, the uncertainty $\sigma_u$ in the calculated value for a function such as $u(x,y,z) = x - 2y + 3z$, is given by:

$$\sigma_u = \sqrt{\sigma_x^2 + 4\sigma_y^2 + 9\sigma_z^2}. \quad (7)$$
If quantities are multiplied by each other or divided by each other such as \( u = kxyz \), or \( u = kx/yz \), then the uncertainty in \( u \) is given by

\[
\sigma_u = \bar{u} \sqrt{\left( \frac{\sigma_x}{x} \right)^2 + \left( \frac{\sigma_y}{y} \right)^2 + \left( \frac{\sigma_z}{z} \right)^2}.
\] (8)

where \( \bar{u} \) is the average value of the measured quantities. If a quantity is raised to a power, such as \( u(x,y) = kx^2y \), then the uncertainty in \( u \) is

\[
\sigma_u = \bar{u} \sqrt{4\left( \frac{\sigma_x}{x} \right)^2 + \left( \frac{\sigma_y}{y} \right)^2}.
\] (9)
iMac Operating Instructions

• To open software, a file, or a folder, double click on it with the mouse.
• To select something on a computer means to click on it once with the mouse.
• The Command key is on each side of the space bar.
• To close software: FILE → Quit [Command + Q]
• To close a folder or document, click on the upper left square of the window, or: FILE → Close Window [Command + W]
• To open a new document: FILE → New [Command + N]
• To collapse (minimize) a document, click on the upper right corner of the window.
• To re-size a document, click and drag the bottom right corner of the window.
• To switch between different programs click on the icon in the very top right corner of the desktop. All open software will be displayed. Then simply scroll down to the one you need, and select it.

When you have finished your experiment:

• ALWAYS close all software: FILE → Quit [Command + Q]
  (Note that closing the window is not enough.)
• Close all open folders: FILE → Close Window [Command+W]
• Restart the computer: Special → Restart
• DO NOT TURN OFF THE COMPUTERS!
Graphs
Consider the equation of a line: \( y = mx + b \). When \( b = 0 \), we’re left with the equation \( y = mx \). When you graph “\( y \) vs. \( x \)” and produce a linear graph, the slope, \( m \), of the graph represents a constant of proportionality. Note that a curve does not have a single slope, it has an infinite number of slopes (a constantly changing slope)! At the bottom of each linear graph write the numeric value of the slope, as well as what the slope represents. Graphing \( A \) vs. \( B \) is equivalent to graphing \( y \) vs. \( x \); the resulting line has slope \( m \). This method is used frequently; familiarize yourself with it. When necessary, plot the point (0,0). (It is usually necessary.)

**GRAPHICAL ANALYSIS Instructions**

- The axes must be labeled, including units. Select the column heading. This will highlight the entire column of data. Move the cursor to the field. Type the new label, then tab to the units field; type the relevant symbol for the unit (do not add parentheses).
- To set the parameters of your graph select the following options:

**FILE**

**PAGE SETUP** → Paper: Letter; Scale: 90%; Orientation: Landscape.

**PRINT OPTIONS** → Type all team members’ names, select "Date".
PREFERENCES  →  enter the desired significant figures or
decimal places. Select: “Point Protector”, “Background Grid” and
“Legend”. Select “Regression Line” and “Regression Statistics” or
“Connecting Line” as needed.

DATA  →  DATA OPTIONS  →  select “Use Preferences”.
  → RENAME DATA SET 1  →  type relevant title for Data Set.

GRAPH  →  GRAPH OPTIONS  →  select “Use Preferences”.
  → RENAME GRAPH  →  type relevant title for your graph.

• For scientific notation, type: E (exponent) {e.g., to enter 3.1x10^-3  type:  3 . 1 E – 3}
• At the bottom of your graph write the slope value (numeric and
  algebraic forms).
  {e.g.,  m = 1/R = 0.00633  R = 1/m = 157.978  (unit)}
DIGITAL MULTI-METER

A Digital Multi-Meter, **DMM**, will measure several different quantities. You must determine which function the DMM will perform by selecting the following four items:

- **POWER SOURCE**: Set the switch to DC (Direct Current) or AC (Alternating Current). We use DC in this lab, except for Exp. 15.

- **LEADS**: The black lead should always plug into the "COM" jack. Although the color of the wire covering is irrelevant, it is standard to define the black wire as the common (ground). The red lead will be plugged into the "VΩ" jack when measuring voltage or resistance. Plug the red lead into the "mA" jack when measuring current (the "10A" jack is used for Exp. 22 and Exp. 23).

- **DIAL**: You must select the function the DMM will perform by turning the dial to the appropriate setting, as well as the appropriate scale. Select the lowest value that will accommodate your circuit, e.g., the '20V' setting will measure voltage up to 20 volts.

- **CONNECTION**: You must connect the leads to your circuit properly, or the DMM could be damaged. A voltmeter is always connected in parallel; the leads connect to each end of the same element. An ammeter is always connected in series; a jumper must be removed from the circuit in order to place the ammeter into the circuit.
Fig. 1: Digital Multi-Meter (DMM)

Note the multipliers $\mu$, $m$, $K$, and $M$ on the different scales. This means you would multiply the digital reading of the DMM by the appropriate multiplier. For example, the 200K scale for the ohmmeter means you should multiply the digital readout by $10^3$.

Note also that the DMM will only read values up to the scale value. For example, the $\frac{20m}{10A}$ scale for the ammeter means the DMM will measure current up to 20mA if using the mA jack, but will measure up to 10A if using the 10A jack.
Fig. 2: **Voltmeter**  (See also Fig. 5.)

Voltmeter Reading: 5.27 V.

To use the DMM as a voltmeter, make these selections:

- **POWER**: Select DC or AC.
- **LEADS**: Plug the black lead into "COM" and the red lead into "VΩ".
- **DIAL**: Turn to the appropriate scale on "V".
- **CONNECTION**: Place leads on each end of *the same element*. 
Fig. 3: **Ohmmeter** (See also Fig. 6 and Fig. 7.)

Ohmmeter Reading: $0.501 \times 10^3 \, \Omega = 501 \, \Omega$.

To use the DMM as an ohmmeter, make these selections:

- **POWER**: Select DC or AC.
- **LEADS**: Plug the black lead into "COM" and the red lead into "VΩ".
- **DIAL**: Turn to the appropriate scale on "Ω".
- **CONNECTION**: Place leads on each end of the same element.
Fig. 4: Ammeter  (See also Fig. 8.)

Ammeter Reading:  $10.2 \times 10^{-3} \text{ A} = 0.0102 \text{ A}$.

To use the DMM as an ammeter, make these selections:
- **POWER**: Select DC or AC.
- **LEADS**: Plug the black lead into "COM" and the red lead into "mA".
- **DIAL**: Turn to the appropriate scale on "A".
- **CONNECTION**: Remove a jumper, place leads into circuit (current travels from one element, through the ammeter, into a different element).
Fig. 5: Voltmeter and Circuit

Voltmeter Reading: 5.01 V.

Note that the circuit is complete, power supply plugged in and set to desired voltage. The voltmeter is connected in parallel, as it measures voltage across an element in a circuit.
Fig. 6: **Ohmmeter and Circuit**  Ohmmeter Reading: 0.501 kΩ=501 Ω.
Note that the power supply has been disconnected in order to measure the resistance, $R_{\text{eq}}$, of the circuit.

Fig. 7: **Ohmmeter and Resistor**  Ohmmeter Reading: 0.200 kΩ=200 Ω.
Connect ohmmeter to resistor for resistance measurement of an individual element.
Fig. 8: **Ammeter and Circuit**  
Ammeter Reading: 9.9 mA = 0.0099 A.  
Note that the ammeter is connected in series. A jumper has been removed from the circuit so that the ammeter can be inserted, in series, to measure the current. Power supply is connected and active.
The Vernier Caliper

Fig. 9: Vernier Caliper

- Note the “zero” tick mark on each of the scales (vernier and main).
- We will always use metric units; ignore British units (inches) on each of the scales (the top of this caliper).
- In our lab, the uncertainty of a measurement, \( \delta \), is defined as \( \frac{1}{2} \) the smallest unit. The precision of the vernier scale is 0.05 mm (equals 0.005 cm). Thus, \( \delta = 0.0025 \) cm for the vernier caliper, allowing a measurement of \( (a.bcde \pm 0.0025) \) cm. We assume a zero for the ten-thousandths place (\( e \)) to accommodate \( \delta \).
- Do not over-tighten the clamp. Tighten just enough that the caliper does not shift.
Fig. 10

- Note that the “zero tick mark” of the vernier scale has passed the 2.2 position, but not the 2.3 position of the main scale. You now have \(a.b \ (2.2)\) of the “\(a.bcd\) cm” measurement.
- To obtain the remaining values, determine the first tick mark (from left to right) on the vernier scale that is aligned with a tick mark on the main scale. In this case, 5.5 is the first alignment, and provides \(c.d\) of your measurement.
- The final measurement reading of the vernier caliper is: 
  \((2.2550 \pm 0.0025)\) cm.
- There is a tutorial on the lab computers:
  
  **Items for Students → YP Vernier 1.1**

Fig. 11
Note that as you open the caliper, a device to measure depth is available.