Experiment 17: Kirchhoff’s Laws for Circuits

Figure 17.1: Kirchhoff’s Law Circuit Board

Figure 17.2: Schematic for Kirchhoff’s Circuit

EQUIPMENT

(1) Universal Circuit Board
(2) D-Cell Batteries (1.5 V)
(2) Battery Holders
(4) Alligator Clips
(1) DMM

(5) Resistors:
$R_1 = 10 \ \Omega \ \text{Resistor}$
$R_2 = 12 \ \Omega \ \text{Resistor}$
$R_3 = 15 \ \Omega \ \text{Resistor}$
$R_4 = 18 \ \Omega \ \text{Resistor}$
$R_5 = 22 \ \Omega \ \text{Resistor}$
Objective
The objective of this experiment is to apply Kirchhoff’s rules for circuits to a two-loop circuit to determine the three currents in the circuit and the electric potential differences around each loop.

Theory
The two basic laws of electricity that are most useful in analyzing circuits are Kirchhoff’s laws for current and voltage.

Kirchhoff’s Current Law (KCL) states that at any junction (node) of a circuit, the algebraic sum of all the currents is zero (sum of the currents entering the junction equals the sum of the currents leaving the junction). In other words, electric charge is conserved.

\[ \sum I_{in} = \sum I_{out} \]  

(17.1)

Kirchhoff’s Voltage Law (KVL) states that around any closed loop or path in a circuit, the algebraic sum of all electric potential differences is equal to zero.

\[ \sum V_i = 0 \]  

(17.2)

To calculate magnitudes of current and voltage in a circuit like Fig. 17.2, you will need to write two equations, making use of KVL and Ohm’s Law. This results in two equations with two unknowns. For this experiment, you will measure \( \varepsilon_1 \) and \( R_i \), then solve for the two currents, \( I_i \).

One might be able to guess the direction of current flow in a circuit, given a circuit such as the one in this experiment. However, the current direction can be safely ignored when using the loop method. For the purposes of this experiment, all currents will be assumed to be clockwise. If any current is measured or calculated to be negative, that current actually flows counterclockwise in the circuit.

Apply the following rules when writing a KVL equation for a loop:

- If a seat of emf is traversed from \(-\) to \(+\), the change in potential is \(+\varepsilon\); if it is traversed from \(+\) to \(-\), the change in potential is \(-\varepsilon\). (Note that you must maintain orientation as you progress around each loop. \( \varepsilon_1 \) is traversed from \(-\) to \(+\), while \( \varepsilon_2 \) is traversed from \(+\) to \(-\).)

- Current flows from high potential to low potential. Crossing a resistance with the current constitutes a negative potential difference. Measuring against the current yields a positive potential difference.

Figure 17.3: Potential Difference Sign Convention

Write an equation for each loop in terms of electric potential difference. The electromotive force, emf \( (\varepsilon, \text{voltage source}) \), is provided by a D-cell battery.

For example, the equation for the left loop in Fig. 17.2 is:

\[ \varepsilon_1 + V_1 + V_2 + V_3 = 0 \]  

(17.3)

Rewrite this equation using Ohm’s Law, then simplify:

\[ \varepsilon_1 - I_1(R_1 + R_2 + R_3) + I_2 R_2 = 0 \]  

(17.4)

A similar equation is written for the right loop. Taken together, the two loop equations can be solved for \( I_1 \) and \( I_2 \) by means of substitution. Theoretical potential differences can then be calculated using Ohm’s law.
Prelab 17: Kirchhoff’s Laws for Circuits

Name: ________________________________

1. Write the equation, then briefly explain: (20 pts ea.)

   (a) Kirchhoff’s Voltage Law (KVL)

   (b) Kirchhoff’s Current Law (KCL)

2. Consider the circuit shown in Fig. 17.2 and the Equipment list on Page 91. Use Kirchhoff’s Voltage Law to solve for the theoretical currents in each of the three branches of the circuit. Let us define two distinct loops of the circuit with currents $I_1$ and $I_2$.

   Voltage sums for the left loop and for the right loop are written using KVL:

   \[
   \varepsilon_1 - V_1 - V_2 - V_3 = 0 \quad \text{(KVL - left loop)}
   \]
   \[
   -\varepsilon_2 - V_4 - V_5 + V_2 = 0 \quad \text{(KVL - right loop)}
   \]

   In performing experiment, measured values will be used for $\varepsilon_i$ and $R_i$. For the pre-lab, use the nominal values as stated in the Equipment list. Recall that the electric potential difference, or voltage, across a resistor has the same subscript as the resistor. For example, the voltage across resistor $R_6$ is $V_6$. Use the back of this sheet for Question 2.

   (a) Rewrite the KVL equations using Ohm’s Law (e.g., $V_1 = I_1 R_1$). (30 pts)

   (b) Solve for $I_1$ and $I_2$ using substitution. Refer to Appendix D as necessary. (30 pts)
**PROCEDURE**

**PART 1: Loop Method - Calculations**

1. Determine the nominal resistance and tolerance of each resistor by reading its color code (Table 15.1, Page 80). They should have the following approximate resistances:
   \[ R_1 = 10 \ \Omega \]
   \[ R_2 = 12 \ \Omega \]
   \[ R_3 = 15 \ \Omega \]
   \[ R_4 = 18 \ \Omega \]
   \[ R_5 = 22 \ \Omega \]

2. Measure the resistance of each resistor using an ohmmeter.

3. Construct the circuit shown in Fig. 17.2. Do not connect the ammeter.

4. Measure \( \varepsilon \) of the two batteries using a voltmeter. They should each be at least 1.1 V. Turn off the DMM and disconnect the batteries so they do not drain.

5. Using your knowledge of Kirchhoff’s Voltage Law, write two equations relating the potential differences across each element in the two loops. Remember that both currents flow through the central branch of the circuit.

6. Solve these equations by substitution to find the theoretical currents in each loop. A negative value simply indicates the current flows in the other direction.

**PART 2: Current & Voltage Laws Applied**

7. Connect the batteries to the circuit.

8. Measure the current in each of the three branches of the circuit. Refer to Fig. 17.4 for proper ammeter connection technique. Disconnect the batteries and turn off the DMM after measurement.

9. Compare the measured values of current with the calculated values. If they are not approximately equal, check your calculations or retest the circuit.

10. Reconnect the batteries and measure the electric potential across each element of the circuit. Sign and direction are crucial; measure clockwise, leading with the red lead and following with black.

**PART 3: Non-Ideal Voltmeter**

At the front of the room, your TA has set up two series circuits. One circuit has two 100 \( \Omega \) resistors, the other circuit has two 10.0 M\( \Omega \) resistors. Take your voltmeter to this table. Adjust the power supply on each circuit to 10.0 V.

11. Measure the potential differences across each of the resistors in the 100 \( \Omega \) circuit. Is the magnitude of their sum equal to the potential difference across the power supply? Show work.

12. Measure the potential difference across each of the resistors in the 10.0 M\( \Omega \) circuit. Is the magnitude of their sum equal to the potential difference across the power supply? Show work.
QUESTIONS

1. Explain what effect the DMM will have on the circuit when inserted to measure current.

2. Do the values from Part 2 verify Kirchhoff’s Current Law?

3. Do the values from Part 2 verify Kirchhoff’s Voltage Law?

4. Would disconnecting the power supply on the left loop of the circuit, $\varepsilon_1$, affect the current $I_2$ (no jumper inserted)? Calculate what $I_2$ is for this case. Comment on $I_3$.

5. A voltmeter is connected in parallel to a resistor when measuring $V_i$. Remember that the internal resistance of the DMM, when used as a voltmeter, is approximately 10 M$\Omega$. Calculate $R_{eq}$ of the voltmeter connected to a resistor in each circuit of Part 3.

6. Use the above information to discuss why the voltage measured across the 10 M$\Omega$ resistors did not equal the voltage measured across the power supply.