Experiment 4
Projectile Motion

Equipment

- PASCO ballistic Pendulum (spring gun)
- two-meter stick
- piece of carbon paper
- meter stick
- ruler
- *plumb bob*

Objective

The objective of this experiment is to measure the speed at which a projectile leaves a spring gun and to predict the landing point when the projectile is fired at a nonzero angle of elevation.

Theory

Projectile motion is an example of motion with constant acceleration. In this experiment, a projectile will be fired from some height above the floor and the position where it lands will be predicted. To make this prediction, one needs to know how to describe the motion of the projectile using the laws of physics. The position as a function of time is

$$\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}_0 t + \tfrac{1}{2}\mathbf{a}t^2. \qquad (1)$$

By measuring appropriate quantities, one can predict where the projectile will strike the floor. Eq. (1) is a general form describing the position of an object. It can be resolved into x and y components as

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2 \qquad (2)$$

and

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 \qquad (3)$$

which give the position of the projectile in the x and y directions. The x and y components of the initial velocity are (Fig. 4-1)

$$v_{ox} = v_0 \cos\theta_0 \quad \text{and} \quad v_{0y} = v_0 \sin\theta_0. \qquad (4)$$

For a projectile, there is no horizontal component of acceleration after the gun is fired. The only acceleration is due to the gravitational attraction of the earth. This acceleration has magnitude g acting in the negative vertical direction (Fig. 4-1). Hence, the Eqs. (2) and (3) become

$$x = x_0 + v_{0x}t \qquad (5)$$

and

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2 \qquad (6)$$

These equations of motion describe the motion of a projectile.

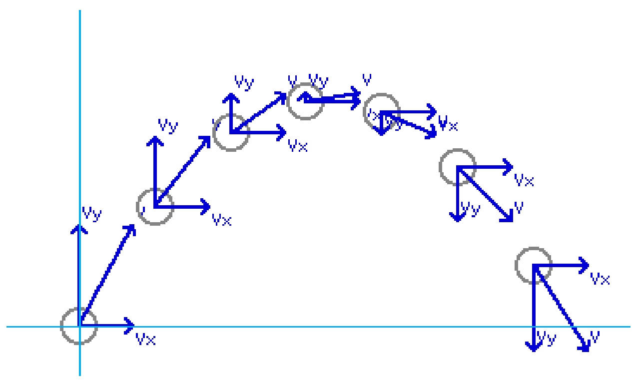

Fig. 4-1 Projectile motion. The trajectory is a parabola.

The range equation describes the horizontal range that a projectile will travel if projected with initial velocity v at an angle θ and gravity is the only force acting the projectile.

$$Range = \frac{v^2 \sin 2\theta}{g} \qquad (7)$$

The equation above assumes the launch point and landing point are both on the same horizontal plane

Procedure

1. Record the number written on the firing mechanism of the ballistic pendulum. You may need it for a future experiment.

2. Place the ballistic pendulum on a platform with pendulum arm in the up position (Fig. 4-1). Measure the height from the floor to the *bottom* of the ball as follows: Use a ruler to measure the distance from the table top to the bottom of the ball and the meter stick to measure the height of the table above the floor. Add these two distances together to get the total height h.

3. Calculate the amount of time the ball will be in the air when fired horizontally. Recall that if two balls are released at the same time, one falling vertically and the other projected horizontally, *both will hit the ground at the same time.*

4. Fire the spring gun from the 3rd detent. (**Be sure that no one is in the flight path!)**. Note where the ball lands and **tape a sheet of white paper** at that spot. Put a piece of carbon paper (i.e., black paper) on top of white paper). *Do not put tape on carbon paper!* Fire the spring gun 3 times.

5. For each trial, measure the total distance the ball traveled horizontally. (Be sure to measure the total distance from **ball diagram** on apparatus to the point of impact on the floor). Find the average horizontal distance of all the trials. From this value and the time calculated in step 3, calculate the speed at which the ball leaves the gun.

6. Reconfigure the ballistic pendulum to shoot from an angle of 45 degrees. Measure the total vertical distance from the bottom of the ball to the platform (board) as well as horizontal distance to edge of platform (board).

7. Using Equation 6 and the quadratic equation calculate the time the ball stays in the air (when fired at 45 degrees). Then use equation 5 to find the horizontal range of the ball when fired at the above angle and height. in step 6.

8. Mark the floor at the location the ball is calculated to land. Place the line on the target at this location. Fire the ball at the target three times and determine the average distance.

Calculate the percentage difference between the range R and the average measured range. If the distance calculated and the distance obtained from firing the gun are substantially different, check your calculations. Fire the gun again after locating and correcting your errors.

9. From the range equation we know that maximum range for a projectile is 45 degrees (if air resistance is ignored). Determine if this is true for the configuration that you are using (i.e., shooting to a point below the horizontal starting point). In other words try other angles and see if you can get the projectile to travel farther.

Post Lab Questions (You must show all work to receive credit)

1. Using methods of The Calculus show that the maximum range of a projectile (using the range equation) is obtained when the angle 45 degrees. This is a max-min type problem from Cal one.

This is done as follows: Take the derivative of range equation with respect to angle, set equal to zero and solve for theta.

2. If the ball had twice the mass, but left the spring gun at the same speed, what effect would this have on its distance of flight? Neglect air resistance. How much more (kinetic) energy would you have to give the ball so that it had the same velocity.

3. Take the sine (in degrees) of (10x2) & then (80x2). What do you notice? Take the sine of (30x2) & then (60x2).

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What does this tell you about the range obtained for the second pair of angles. Draw a diagram to demonstrate.

4. Discuss what you observed when you changed the angle from 45 degrees to some other angle (i.e., was there some other angle other than 45 degrees which gave you a maximum value.) What do you think would happen to this angle as the distance to the floor (ground) approached infinity?

5) In this experiment the quadratic equation was used to calculate the length of time the ball was in the air when the angle was 45 degrees. There were two solutions of the quadratic equation, one positive and the 2nd solution was negative.

Although you used the positive answer (root) the negative root has a physical meaning. It is the time it takes for a projectile fired (from the floor to reach the table) if it follows the same path as the ball shot from the table.

This means that the ball has an initial velocity greater than 5 m/s when fired and it slows to 5 m/s by the time it travels one meter in height and reaches the table.

You are to show that this is the case.

The angle at the table is 45 degrees and
the height above the table is one meter:

a) Calculate the time the ball stays in the air (done the same as in lab). Initial velocity at the height of the table is 5 m/s.

b) Using the y component only, calculate the velocity $v_{y\,\text{floor}}$ that a ball fired from the floor would have if it had traveled one meter from the floor to the table and had a y velocity of v_{0y} when it reached the table. Use the following formula.

$$v_{y\,\text{floor}}^2 = v_{0y}^2 + 2a(\Delta y)$$

c) Using the velocity $v_{y\,\text{floor}}$ from part b) calculate the maximum height the ball would travel. (The velocity is zero at the top of trajectory). *Be careful of signs.*

d) Use $\Delta y = \frac{1}{2}at^2$ to calculate the time the ball stays in the air. Remember to multiply by 2 for total time in the air.

e) Compare the time in part a) to the time in part d). Is the difference equal to the negative root obtained in part a)?