## Uncertainty calculations worksheet for Moments of Inertia

## You will need to fill in the boxes below and turn this worksheet as sample calculations for uncertainty.

The following notes are arranged to give the student step by step guidance on the uncertainty calculations for the moment of inertia experiment. The "step \#"refers to the experiment procedure step number. The gravitational constant g is assumed to be $(9.8000 \pm 0.0001) \mathrm{m} / \mathrm{s}^{2}$ (for our experiments this uncertainty in g is negligible.) Be sure and convert all units to SI units

## Uncertainty for Step 2- frictional torque.

Because you only qualitatively determined (by sight) "zero angular acceleration" you established a range of masses which gave you a minimum negative acceleration to minimum positive acceleration. It is generally easier to see these accelerations than it is to see zero acceleration. The average of these two masses will be used as a best estimate of friction mass. Thus friction mass and its uncertainty is given by

$$
m_{\text {friction average }}+\delta m_{\text {friction }}=\frac{\left(m_{\text {friction high }}+m_{\text {friction low }}\right)}{2} \pm \frac{\left(m_{\text {friction } \text { high }}-m_{\text {friction low }}\right)}{2}=
$$



Since the uncertainty of the friction mass above (typically 0.5 to 1.0 g ) is about 50 to 100 times larger than the uncertainty of mass given by a balance (i.e., 0.01 g assuming you measured your masses with a balance) we will ignore this uncertainty in this step.

All of that said, frictional tension $T_{f}$ and its uncertainty $\delta T_{f}$ is given by
Eq-1 $\left.\quad T_{f} \pm \delta T_{f}=g\left(m_{\text {friction }} \pm \delta m_{\text {friction }}\right)=g m_{\text {friction }} \pm g \delta m_{\text {friction }}\right)=$ $\square$
Frictional torque $\tau_{\text {fricion }}$ is given by $\tau_{\text {friction }}=$ radius $\times$ Force $=r T_{\text {fricion }}$ and thus the uncertainty in torque is given by

Eq-2

$$
\delta \tau_{\text {friction }}=\tau_{\text {friction }} \sqrt{\left(\frac{\delta r}{r}\right)^{2}+\left(\frac{\delta T_{\text {friction }}}{T_{\text {friction }}}\right)^{2}}=
$$

$\square$

## Uncertainty for Step 4- acceleration \& angular acceleration

The uncertainty in acceleration $\delta a$, where $a=2 d / t^{2}$, is given by

Eq-3

$$
\delta a=a \sqrt{\left(\frac{\delta d}{d}\right)^{2}+4\left(\frac{\delta t}{t}\right)^{2}}=
$$

The uncertainty in angular acceleration $\delta \alpha$, where $\alpha=a / r$, is given by
Eq-4 $\quad \delta \alpha=\alpha \sqrt{\left(\frac{\delta a}{a}\right)^{2}+\left(\frac{\delta r}{r}\right)^{2}}=$

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## Uncertainty for step 5-string tension (assume all masses have a one percent uncertainty)

String tension is given by $T=m(g-a)=m g-m a$. Thus, you will first determine $\delta(m g)$ and $\delta(m a)$ and then add the two uncertainties in quadrature.

Eqs $5 \& 6 \quad \delta(m g)=\mathrm{g} \delta \mathrm{m} \quad$ and $\quad \delta(m a)=\mathrm{ma} \sqrt{\left(\frac{\delta m}{m}\right)^{2}+\left(\frac{\delta a}{a}\right)^{2}}=$
The uncertainty $\delta T$ for the sum (actually difference) of the tension is given by

Eq-7

$$
\delta T=\sqrt{[\delta(m g)]^{2}+[\delta(m a)]^{2}}=
$$

Uncertainty for step 6- torque and net torque $\square$
The uncertainty in torque $\delta \tau$ is given by
Eq-8 $\quad \delta \tau=\tau \sqrt{\left(\frac{\delta T}{T}\right)^{2}+\left(\frac{\delta r}{r}\right)^{2}}=$
The uncertainty, $\delta \tau_{\text {net }}$, in net torque, $\delta \tau_{\text {net }}=\tau-\tau_{\text {friction }}$, is given by

Eq-9

$$
\delta \tau_{\text {net }}=\sqrt{(\delta \tau)^{2}+\left(\delta \tau_{\text {friction }}\right)^{2}}=\square
$$

## Uncertainty for step 8- moment of inertia

The experimental moment of inertia is given by $I_{0 \text { exp }}=\tau_{\text {net }} / \alpha$ and the uncertainty, $\delta \alpha$, is given by

Eq-10

$$
\delta I_{0 \exp }=I_{0 \exp } \sqrt{\left(\frac{\delta \tau_{n e t}}{\tau_{n e t}}\right)^{2}+\left(\frac{\delta \alpha}{\alpha}\right)^{2}}=
$$

$\square$
Uncertainty for step 9- additional masses (assume masses have one percent uncertainties)
The relationship for additional masses is given by $I_{\text {cal }}=I_{0 \text { exp }}+M R^{2}$, where the uncertainty, $\delta\left(M R^{2}\right)$ in the term $M R^{2}$, is given by

Eq-11

$$
\delta\left(M R^{2}\right)=M R^{2} \sqrt{\left(\frac{\delta M}{M}\right)^{2}+4\left(\frac{\delta R}{R}\right)^{2}}=
$$

The uncertainties $\delta\left(M R^{2}\right)$ and $\delta I_{\text {exp }}$ are now added in quadrature to obtain $\delta I_{c a l}$. Show this below

