

## PHYS 221 Measurement

### Uncertainty example using simple propagation of uncertainty rules

The following error propagation (sample calculations) consists of the ‘simple’ methods outlined in lab appendix (pages A7-A9). This method yields uncertainties **which are slightly high, but still gives ‘reasonably good values’**.

For **added/subtracted quantities**, the uncertainties are obtained (propagated) by simply **adding the absolute uncertainties** (*i.e., they are not added in quadrature*).

- Write correct significant figures based on the final uncertainty.

For **multiplied/divided quantities**, the uncertainties are obtained by 1) **converted** to percent uncertainties (*i.e., fractional uncertainties*), and 2) the percent uncertainties are **simply added** (*i.e., they are not added in quadrature*).

- Convert from percent to absolute uncertainties (to get correct significant figures for final answer).

**Important note for uncertainty calculations –Keep extra significant figures in uncertainties when doing computations. Convert to one significant figure in the final number (*i.e., final answer*)!!!**

**Sample Calculations for uncertainty of a volume** (using simple method estimation of uncertainty propagation)

**Volume of block** (a cuboid) from lengths measured using vernier caliper:

$$V_{\text{metal}} = lwh = (2.540 \pm 0.005)\text{cm} \times (5.080 \pm 0.005)\text{cm} \times (7.620 \pm 0.005)\text{cm} \Rightarrow (98.32238 \pm \delta V_{\text{metal}})\text{cm}^3$$

**Convert to percent (fractional uncertainties)**

$$V_{\text{metal}} = lwh = (2.540\text{cm} \pm 0.197\%) \times (5.080\text{cm} \pm 0.0984\%) \times (7.620 \pm \text{cm} \pm 0.0656\%)$$

*Approximate percent uncertainty of volume  $\delta_{\%} V_{\text{metal}}$  is obtained by simple addition of uncertainties, *i.e.,**

$$\delta_{\%} V_{\text{metal}} \approx 0.197\% + 0.0984\% + 0.0656\% = 0.361 = 0.4\%$$

Thus we see  $V_{\text{metal}} = lwh = (98.32238\text{cm}^3 \pm 0.4\%)$

## PHYS 221 Measurement

### Uncertainty example using simple propagation of uncertainty rules

**Convert to absolute uncertainties to get correct number of significant figures.**

We see 0.4% of  $98.32238\text{cm}^3$  is

$$98.32238\text{cm}^3 \times 0.4 / 100 = 98.32238\text{cm}^3 \times 0.004 = 0.393\text{cm}^3 = 0.4\text{cm}^3$$

Thus we see  $V_{\text{metal}} = lwh = (98.3 \pm 0.4)\text{cm}^3$

Mass of metal block obtained from triple beam balance (given in absolute and fractional uncertainties):

$$M_{\text{metal}} \pm \delta M_{\text{metal}} = (450.90 \pm 0.05)\text{g} \quad \Rightarrow \quad M_{\text{metal}} \pm \delta_{\%} M_{\text{metal}} = 450.90\text{g} \pm 0.01\%$$

**Now we need to calculate the density and its uncertainty**

$$\rho_{\text{metal}} = \text{mass} \div \text{volume} = (450.90\text{g} \pm 0.01\%) \div (98.3\text{cm}^3 \pm 0.4\%) = 4.5869\text{g} / \text{cm}^3 \pm \delta\rho_{\% \text{metal}}$$

Now we see the (approximate) fractional uncertainty in density of metal block,  $\delta\rho_{\text{metal}}$  is

$$\delta\rho_{\% \text{metal}} \approx (0.01\% + 0.4\%) = 0.411\% = 0.4\%$$

The density of the block is

$$\rho_{\text{metal}} = 4.5869\text{g} / \text{cm}^3 \pm 0.4\%$$

Lastly, convert to absolute uncertainty to get correct number of significant figures, i.e., 0.4% of 4.5869 is

$$\rho_{\text{metal}} = 4.5869\text{g} / \text{cm}^3 \times 0.004 = 0.018\text{g} / \text{cm}^3 = 0.02\text{g} / \text{cm}^3$$

Finally, the density (with the correct number of significant figures is)

$$\rho_{\text{metal}} = 4.59 \pm 0.02\text{g} / \text{cm}^3$$