PHYS 733: Elementary Particle Physics **Spring 2018**

Homework #4 **Due Date: 4/10/18**

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1. In the center-of-mass frame for the process $\mathrm{AB} \to \mathrm{CD}$, show that

$$dQ = \frac{1}{4\pi^2}\frac{p_f}{4\sqrt{s}}d\Omega$$

$$F = 4p_i\sqrt{s}$$

And hence that the differential cross section is

$$\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{1}{64\pi^2 s}\left(\frac{p_f}{p_i}\right)|\mathcal{M}|^2$$

2. Show that for very high-energy "spinless" electron-muon scattering,

$$\left.\frac{d\sigma}{d\Omega}\right|_{cm} = \frac{\alpha^2}{4s}\left(\frac{3+\cos\theta}{1-\cos\theta}\right)^2$$

Where θ is the scattering angle and $\alpha = e^2/4\pi$. Neglect the particle masses.

3. Show that in the reaction $A + B \to C + D$,

$$s + t + u = m_A^2 + m_B^2 + m_C^2 + m_D^2$$

4. Taking $e^+e^- \to e^+e^-$ to be the *s* channel process, verify that

$$s = 4(k^2 + m^2)$$

$$t = -2k^2(1-\cos\theta)$$

$$u = -2k^2(1+\cos\theta)$$

Where θ is the center-of-mass scattering angle and $k = |\mathbf{k}_i| = |\mathbf{k}_f|$, where $\mathbf{k}_i$ and $\mathbf{k}_f$ are, respectively, the momenta of the incident and scattered electrons in the center-of-mass frame. Show that the process is physically allowed provided $s \geq 4m^2, t \leq 0, u \leq 0$. The physical region is shown shaded in the figure drawn in class. Note that $t = 0 (u = 0)$ correspond to forward (backward) scattering.

5. Show that the invariant amplitude for "spinless" electron-positron scattering,

$$-i\mathcal{M}_{e^+e^-} = -i\left(-e^2\frac{(p_A+p_C)_\mu(-p_D-p_B)^\mu}{(p_D-p_B)^2} - e^2\frac{(p_A-p_B)_\mu(-p_D+p_C)^\mu}{(p_C+p_D)^2}\right),$$

can be written as

$$\mathcal{M}_{e^+e^-}(s,t,u) = e^2\left(\frac{s-u}{t} + \frac{t-u}{s}\right).$$

Comment on the symmetry of $\mathcal{M}$ under $s \leftrightarrow t$.

6. In $e^+e^- \to \tau^+\tau^-$ near threshold, one can obviously not neglect the mass of the τ. Working from $\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\left(\frac{p_f}{p_i}\right)|\mathcal{M}|^2$ and the exact spin-averaged amplitude

$$|\mathcal{M}|^2 = \frac{8e^4}{q^4}\left[(p\cdot p')(k\cdot k') + (k'\cdot p)(k\cdot p') + m^2 k'\cdot p' + M^2 k\cdot p + 2m^2M^2\right]$$

k, p : incoming e^+, e^- four vectors

k', p' : scattered τ^+, τ^- four vectors

m, M : masses of e, τ

Show that the total cross-section for τ production is given by

$$\sigma = \frac{4\pi\alpha^2}{3s}\beta\left(\frac{3-\beta^2}{2}\right) \text{ where } \beta = \frac{v_\tau}{c}.$$