## Homework #1

## 1. Problem 1.1

2. The collision of two particles, each of mass M, is viewed in a Lorentz frame in which they hit head on with momenta equal in magnitude but opposite in direction. We speak of this as the "center-of-mass" (CM) frame (though the name "center-of-momentum" would be more appropriate). The total energy of the system is  $E_{cm}$ . Show that the Lorentz invariant

 $s \equiv (p_1 + p_2)_{\mu} (p_1 + p_2)^{\mu} \equiv (p_1 + p_2)^2 = E_{cm}^2$ . If the collision is viewed in the "laboratory" frame where one of the particles is at rest, then show, by evaluating the invariant *s*, that the other has energy  $E_{lab} = \frac{E_{cm}^2}{2M} - M$ . We can see from this result that colliding-beam accelerators have an enormous advantage over fixed-target accelerators in achieving a given total CM energy,  $\sqrt{s}$ . List some advantages of fixed-target accelerators.

3. a) When a 100 GeV π<sup>+</sup> decays to μ<sup>+</sup>v, the energies in the lab system will depend on the decay angle in the CM system. Find E<sub>v</sub>(max) and E<sub>v</sub>(min) in the lab. Numerical values are required.
b) Find the neutrino energy if the decay angle in the lab is 1 mrad (corresponding to the edge of a 2 m target at the end of a 1 km beam line).

c) Repeat both a) and b) for the decay  $K^+ \rightarrow \mu^+ \nu$ .

## 4. Problem 2.1

- 5. Problem 2.4
- 6. Problem 2.5
- 7. Problem 2.6