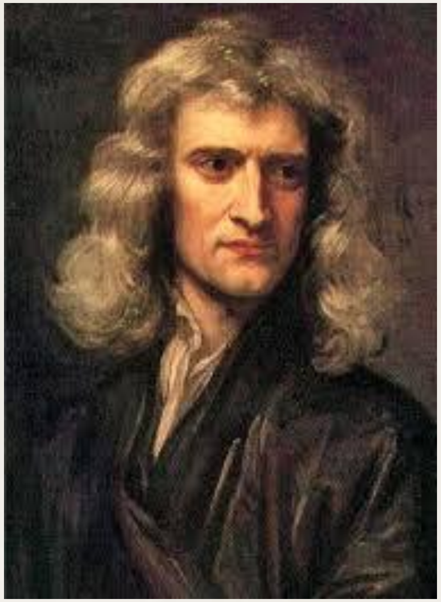
Isaac Newton and the Laws of Motion and Gravitation 1

ASTR 101

10/8/2018

Newton’s laws of motion
Newton’s law of universal gravitation
Orbits, Newton’s cannonball experiment
Mass and weight
Weightlessness in free falls
Center of mass

Isaac Newton



English physicist and mathematician

- One of the most influential physicist of all time.
- Made seminal contribution in mechanics, optics and mathematics.

- Born in Lincolnshire, England, 1643.
- In 1661, Entered Trinity College of Cambridge, graduated in 1665.
- In 1665 the university was closed due to a plague epidemic.
  - Newton spent much of the next two years back at the family farm, reading and thinking and working on his theories on nature.
  - It was during this time he began to think about gravity, made initial discoveries on optics and calculus.
- In 1667 Newton returned to Cambridge as a Fellow. In 1669 he was appointed the Lucasian Professor in mathematics.
- In 1705 he was knighted. By this time Newton had become one of the most celebrated persons of his time.
- In 1727 Newton died and was buried in the Westminster Abbey.

Newton’s laws of Motion

1. An object remains at rest or moves along a straight line with constant speed as long as no net external force acts on it.

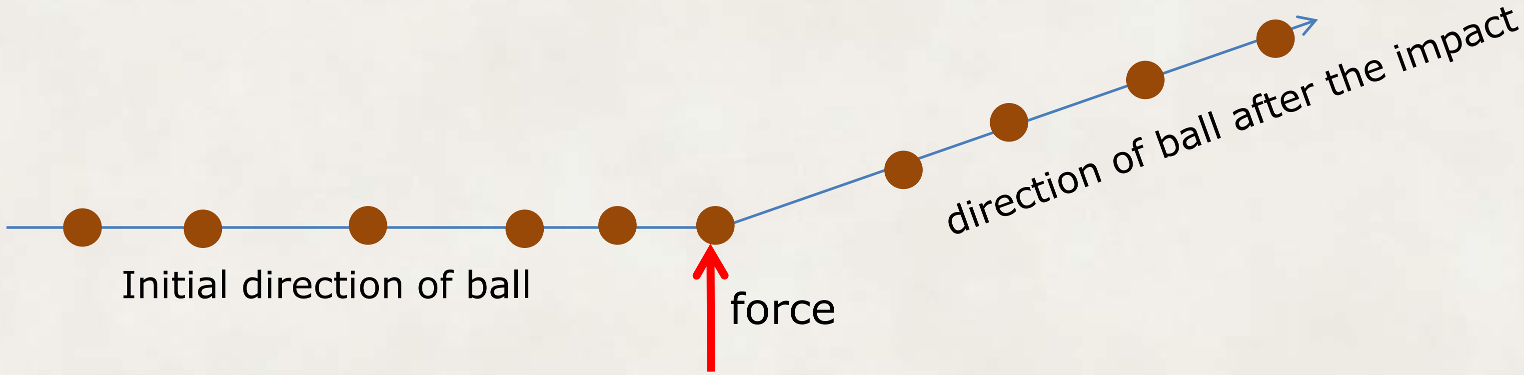
2. When a force is applied on an object, it will accelerate in the direction of the applied force. The acceleration is proportional to the applied force and inversely proportional to its’ mass.

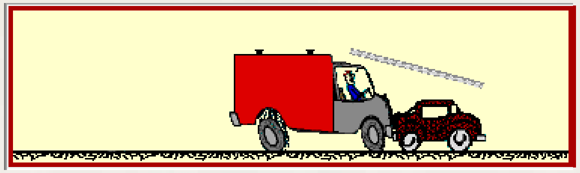
3. To every action, there is an equal and opposite reaction.

First Law

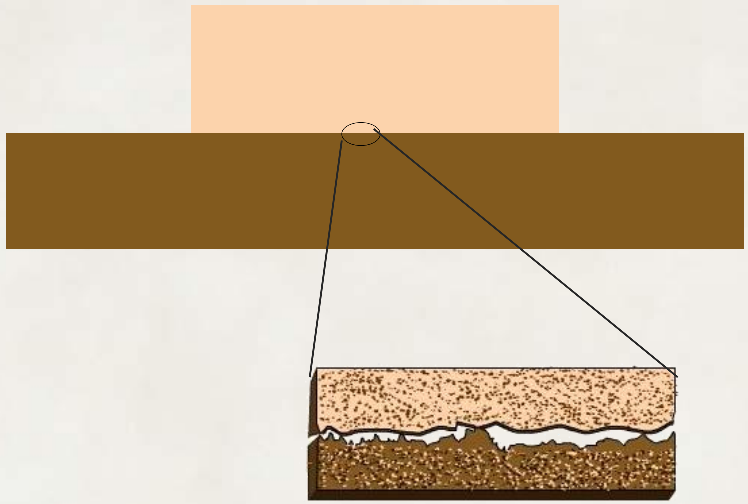
An object remains at rest or moves along a straight line with constant speed as long as no net external force acts on it.

- This implies that all objects have a tendency to resist changes in their state of motion. (to change speed or direction of motion)
- This property of matter, resistance to change its state of motion is called **inertia.** Hence the 1st law is also called the law of inertia.
  - Larger the mass of an object, it has a higher inertia (more resistance to change of motion)





inertia in action



Friction due to surface roughness:
Surfaces have tiny microscopic ridges and bumps, as one surface slides across another, those irregularities catch and lock into each other causing friction.

- Why we do not see this everyday?

  Usually objects in motion slow down and become motionless seemingly without any outside force.

In fact there is a force: **the friction**:

Restraining force generated from the rubbing action between objects, or media (water, air) when there is a contact between them.

Frictional forces act against the motion of an object.

The Second Law

The 2nd law describes how the state of motion of an object changes when a force is applied.

When a force is applied on an object it will accelerate in the direction of the applied force. The acceleration is proportional to the applied force and inversely proportional to its' mass.

- larger force ⇒ higher acceleration.
- Larger mass ⇒ less acceleration

$$a = \frac{F}{m} \quad \Rightarrow \quad F = ma$$

m : mass of the object
F : force
a : acceleration

- Force is measured in Newtons (N):
  - when mass in kilograms, distance is in meters, and time is in seconds in the above formula, the force is given in Newtons (N)

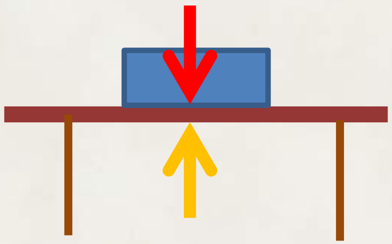
Weight of 1 kg object is about 10 Newtons

Weight of 1 lbs is about 4.5 N

Third Law

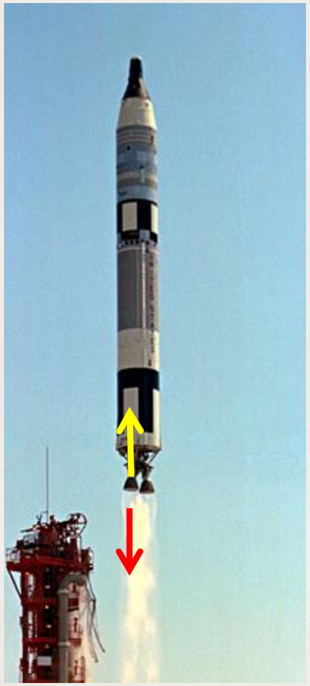
To every action, there is an equal and opposite reaction

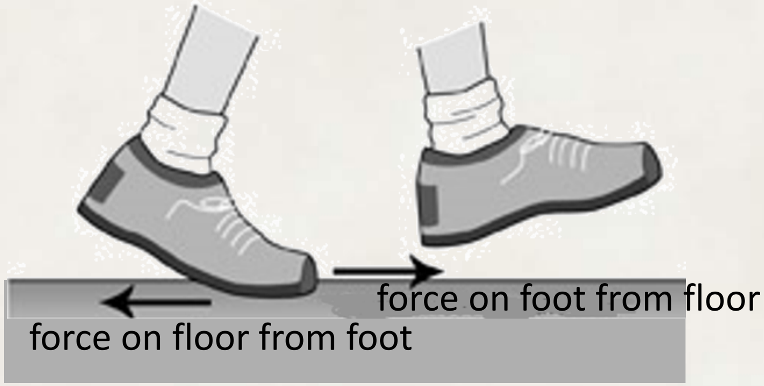
This simply says that interaction forces comes in pairs, and the action and reaction are of the same magnitude in opposite directions.



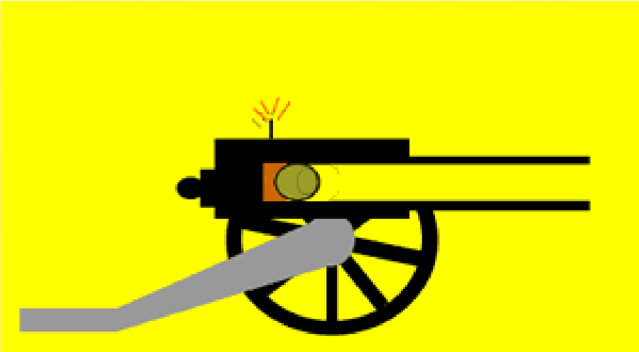
When an object is sitting on a table, weight of the object pushes the table down, and table pushes the object upward equally hard.

When rowing a boat one pushes water backward with oars. And water pushes the oars (and the boat) forward.



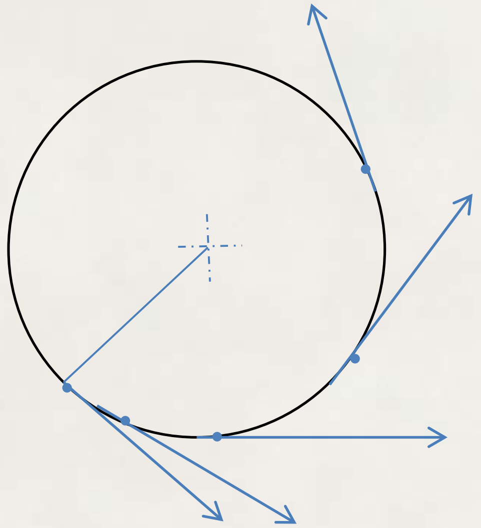


A rocket pushes burned gases down, gas pushes the rocket up by an equal force.



Gun pushes burned gas (and the cannonball) forward, gas in turn push the gun backward.

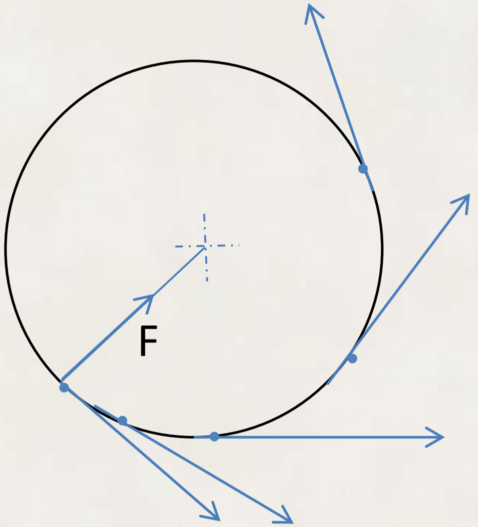
Circular motion:



- When an object is moving in a circle, every moment its direction of motion is changing.
- So we know (from 1st law) it has to be under influence of some force.
  - example: when one swings a tethered ball in a circle one feels the tension of the string.
    - That is because natural tendency of the ball is to move in a straight line.
    - In a circular motion the ball is always changing its direction towards the center.
    - To move the ball in a circular path, a force has to be applied to pull the ball towards the center from the straight path of motion ball is trying to maintain.
    - The tension of the string provides that force.
- The force needed to move an object in a circular (or curved) path is called the **centripetal force**

Circular motion:

- If a force is needed to keep an object move in a circle, is it accelerating?
  - Yes, an objects moving in a circle is always accelerating towards the center.



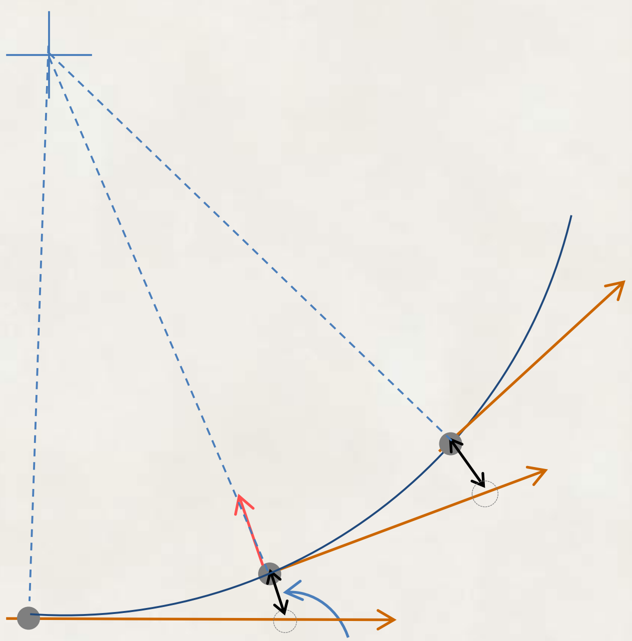
acceleration towards the center $= \dfrac{v^2}{r}$

v: speed of the object, r: radius of the circle

The force needed to move in a circular path

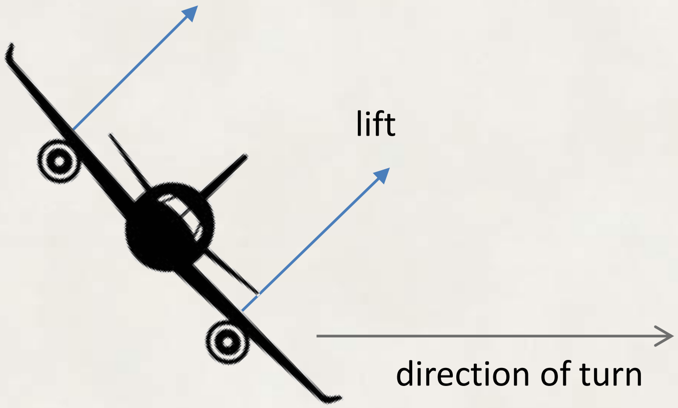
centripetal force $$F = \frac{mv^2}{r}$$

m: mass of the object



Centripetal Force: Examples

In order to follow a curved path, an inward force, the centripetal force is needed.

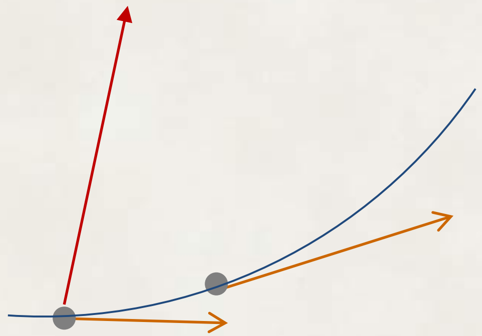


When turning, an aircraft banks the wings in the direction of the turn. Part of the lift from wings provides the needed centripetal force

Roads are banked at turns to provide centripetal force

2013 Spanish train crash https://youtu.be/8N0pMDO4hsQ

Speed and Velocity

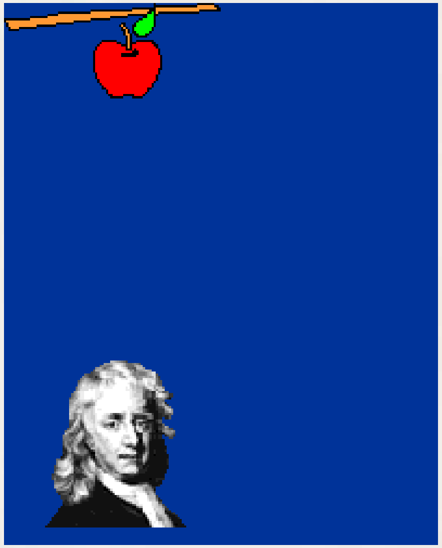


- A force is needed to change direction of motion as well as speed.
- Speed alone does not fully describe the motion of an object.
- **Velocity** is defined as the speed taken with the direction of motion.
  - To specify velocity magnitude of rate of change (speed, a number), as well as the direction of motion are needed.
  - Circular motion is an example where there could be a constant speed, but changing velocity.
- The fist law: "An object remains at rest or moves along a straight line with constant speed as long as no net external force acts on it"

  can be expressed in terms of velocity as:

  "An object remains at rest or moves with constant velocity as long as no net external force acts on it"

Gravitation



Woolsthorpe manner and the Apple tree today

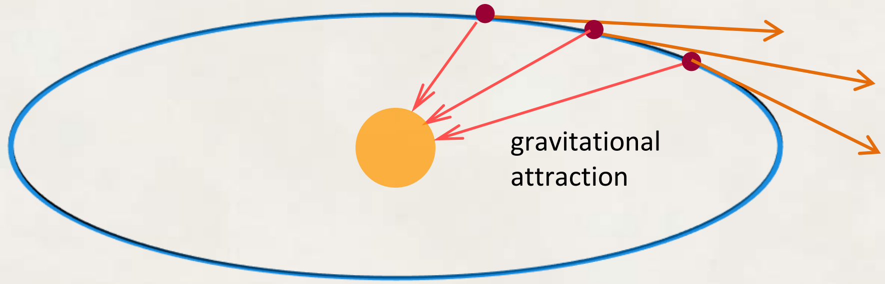
- The popular story says that, Newton was sitting under an apple tree in the garden at Woolsthorpe. An apple fell on his head, and in a stroke of brilliant insight, he came up with the theory of gravitation.

(more about Newton's discovery and the apple at:
www.independent.co.uk/news/science/the-core-of-truth-behind-sir-isaac-newtons-apple-1870915.html)

- In reality, it was an idea Newton developed over time. Probably he began to think about it after seeing an apple falling from a tree in his mother's garden.
  - He had also been trying to understand the orbit of the Moon those days.

He may have thought like this:

- There is a force pulling everything towards the Earth, that is why apple (and other objects) fall.
- The Moon goes around the Earth. There has to be a force pulling the Moon towards the Earth for it to keep doing that. (otherwise moon would move away from Earth).
- The Moon is located at a greater distance, does the force which causes apples to fall on ground extends up to the Moon and keeps it moving around the Earth?
- Newton showed in fact it does.
  - The force which pulls things to ground is the same force that pulls the Moon towards the Earth and keeps it going around the Earth. (just like the tension of a string keeps a ball in a circular path)

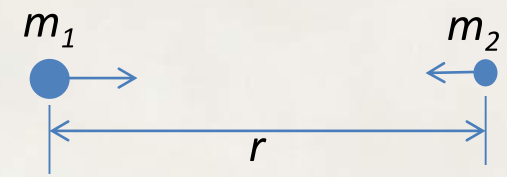


- Thus Newton realized that all objects in the universe orbit around each other due to an attractive force between them, the **Gravitational** force
  - The Earth's gravitational attraction on the moon provides the centripetal force needed to keep Moon going around the Earth.
  - In the same way planets must be pulled by the Sun's gravitational attraction towards the Sun, which keep them moving around the Sun.
- This was a major revelation:
  - The force pulls us to the ground, or causes apples to fall is the same force which holds the whole solar system (and the universe) together and causing them to move in orbits around each other.
  - Since ancient times various ideas had been suggested what holding the Sun Moon and planets in the sky and keep them moving.
- Finally Newton found the answer. He expressed his findings in the law of universal gravitation.
- In 1916 Albert Einstein explained what causing this attraction in his general theory of relativity.

Newton's law of universal gravitation

Any two objects in the universe attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

gravitational force $F = G\dfrac{m_1 m_2}{r^2}$



F : gravitational force between two objects

m_1 : mass of the first object

m_1 : mass of the second object

r : distance between the two objects

G : gravitational constant

$$G = 6.67 \times 10^{-11}\, Nm^2 kg^{-2}$$

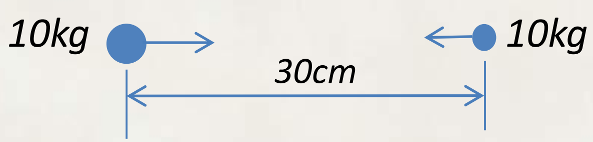
(force in Newtons, distances in meters and mass in kilograms)

- Gravitational attraction between ordinary objects is very weak.
  - Example: gravitational attraction between two iron balls each 10kg, at a distances of 30 cm (1 foot):

m_1 =10 kg, m_2 =10 kg, r= 0.3 m, G=6.67x10^{-11}Nm^2kg^{-2}

$$F = G\frac{m_1 m_2}{r^2}$$

$$= 6.67\times10^{-11}\frac{10\times10}{(0.3)^3} \quad = 7.4\times10^{-8}\,N$$



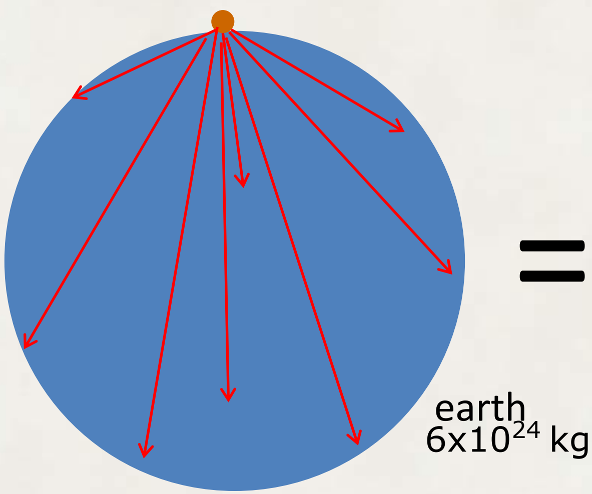
which is extremely small! (cf. 1 *Newton* is about the weight of ¼ lbs)

But on cosmic scales gravitational force is significant because objects are massive, eg. Earth 6x10^{24} kg, Sun, 2x10^{30} kg

But on cosmic scales gravitational force is significant because objects are massive, eg. Earth $6\text{x}10^{24}$ kg, Sun, $2\text{x}10^{30}$ kg

For example a 1kg object on Earth: its attraction towards the Earth is significant because the mass of the Earth is large.

m_1 =1 kg,   mass of earth: m_1 = $6\text{x}10^{24}$ kg,
radius of earth r= 6380 km=$6.38\text{x}10^{6}$ m



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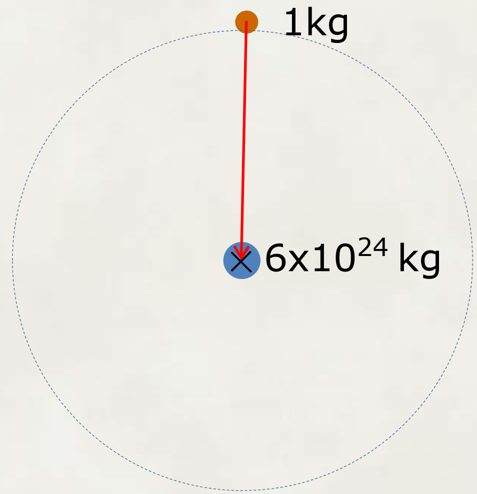


Matter from different parts of the earth are at different distances from the object.
They pull on the object by different strengths according to the gravitational law.
(stronger pull from nearby matter than matter far away)

Total gravitational force is equivalent to the pull of the mass of the whole earth at the center.

Therefore Earth's gravitational pull on the object

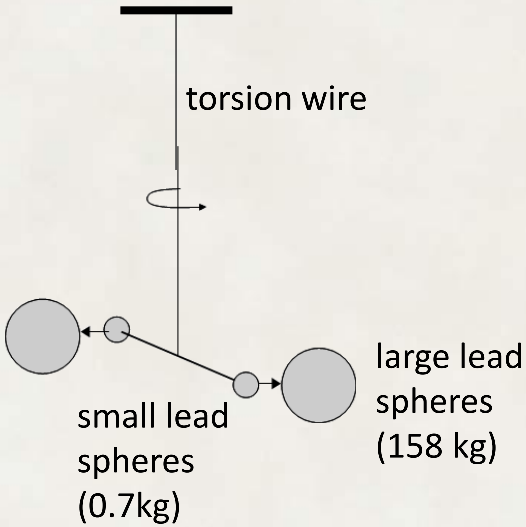
$$F = G\frac{m_1 m_2}{r^2}$$

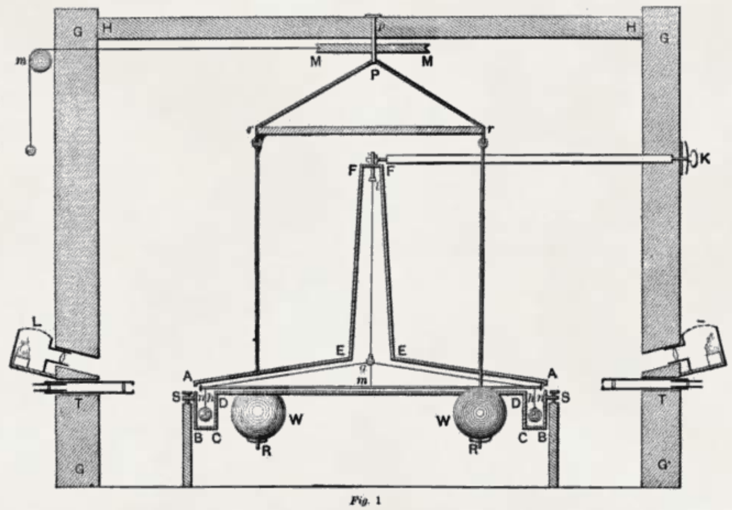
$$= 6.67\times 10^{-11}\frac{2\times 10^{24}\times 1}{\left(6.37\times 10^{6}\right)^2}$$

$$= 9.8N$$

$$a = \frac{F}{m}$$

It is called the weight of the object.



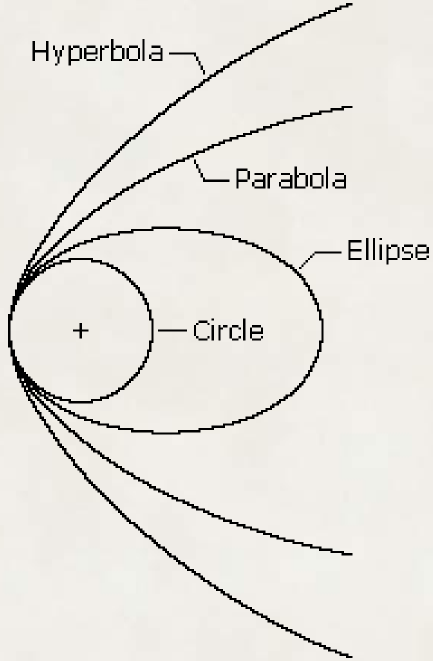


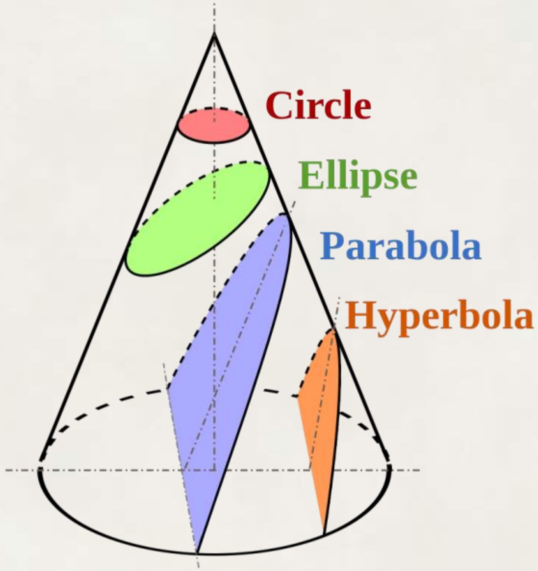
Cavendish’s original apparatus

- Though very weak, the gravitational attraction between two objects can be measured in sensitive experiments.
  - This was how the value of the gravitational constant *G* was first measured by the British physicist Henry Cavendish in 1798 using a torsion balance.
- Once the value of *G* was known, using the gravitational formula, it was possible to estimate the mass of the Earth, the Sun or anything which had an object orbiting around it.

$$F = G\frac{m_1 m_2}{r^2}$$

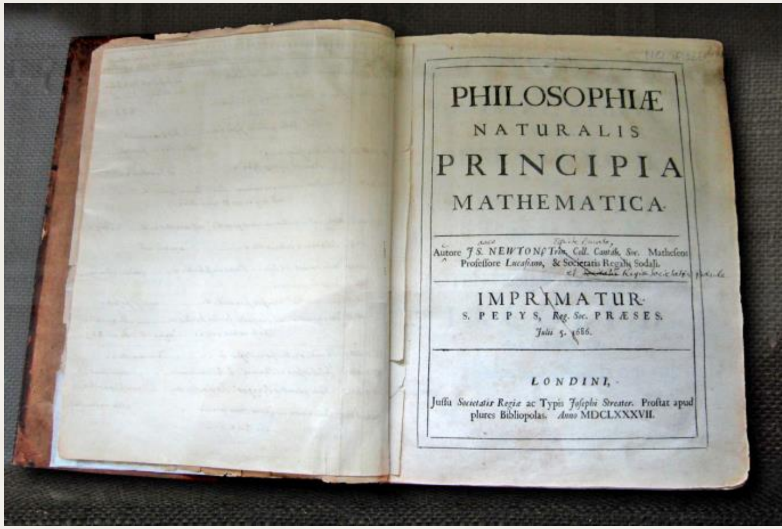
Planetary orbits and Conic Sections

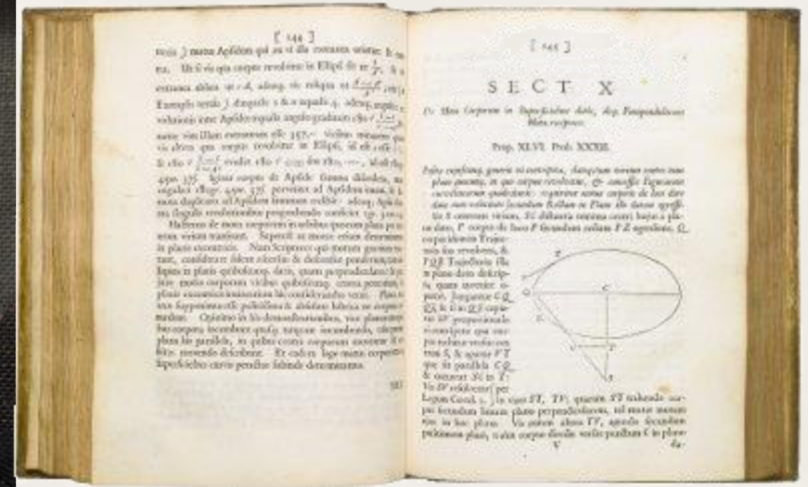




- Newton showed that it was possible to mathematically deduce all three of Kepler's laws of planetary motion according to his theory of gravitation.
- A complete mathematical analysis showed that orbits could generally be **conic sections** (circular, elliptical, parabolic or hyperbolic orbits).
- Circle and ellipse are bounded (enclosed) curves, a planet orbiting on such orbit continues to go around the sun.
- Parabola and hyperbola are unbounded (open) curves extend to infinity. An object on such orbit eventually leaves the solar system.
  - Some comets follow such orbits, so they are seen only once, do not reappear.

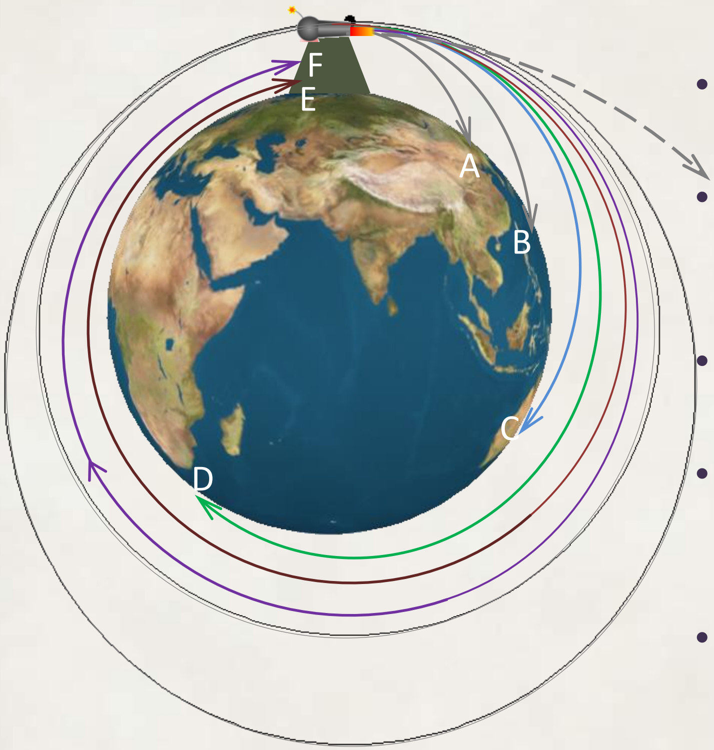
Principia





- Laws of motion and gravitation were published in Newton's book *"Mathematical Principles of Natural Philosophy"* or ***Principia*** in 1687.
  - It is regarded as one of the most important works in the history of science.
  - It was in three books.
    - Book I: Laws of motion
    - Book II: Hydrostatics and hydrodynamics,
    - Book III: Law of universal gravitation.
- As the story goes, Edmund Halley visited Newton in 1684 and asked what would be the shape of a planetary orbit, if the force between it and the Sun changes as inverse square of the distance.
- Newton had replied that it would be an ellipse, and that he had worked it out years ago. He had promised Halley to send the details.
- Newton later reworked the proof for Halley, in the course of which he began developing a book-length discussion on the subject of motion of bodies in orbit. This was what written out into Principia during 1685-1686.

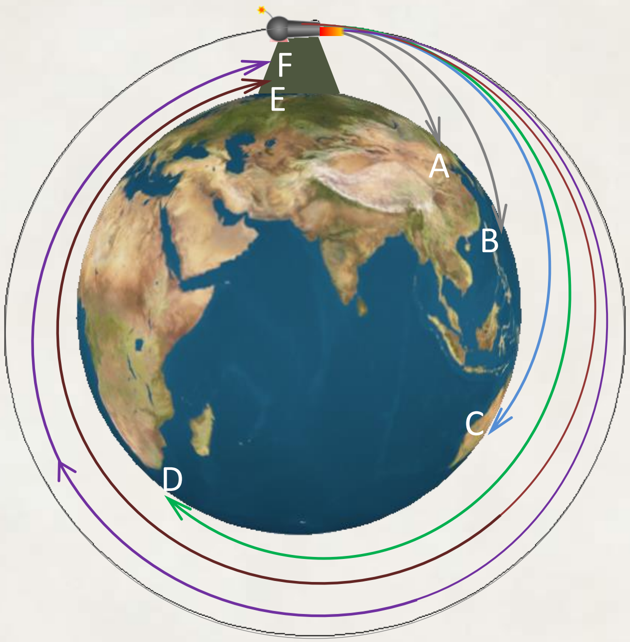
Newton's cannonball experiment - Orbits

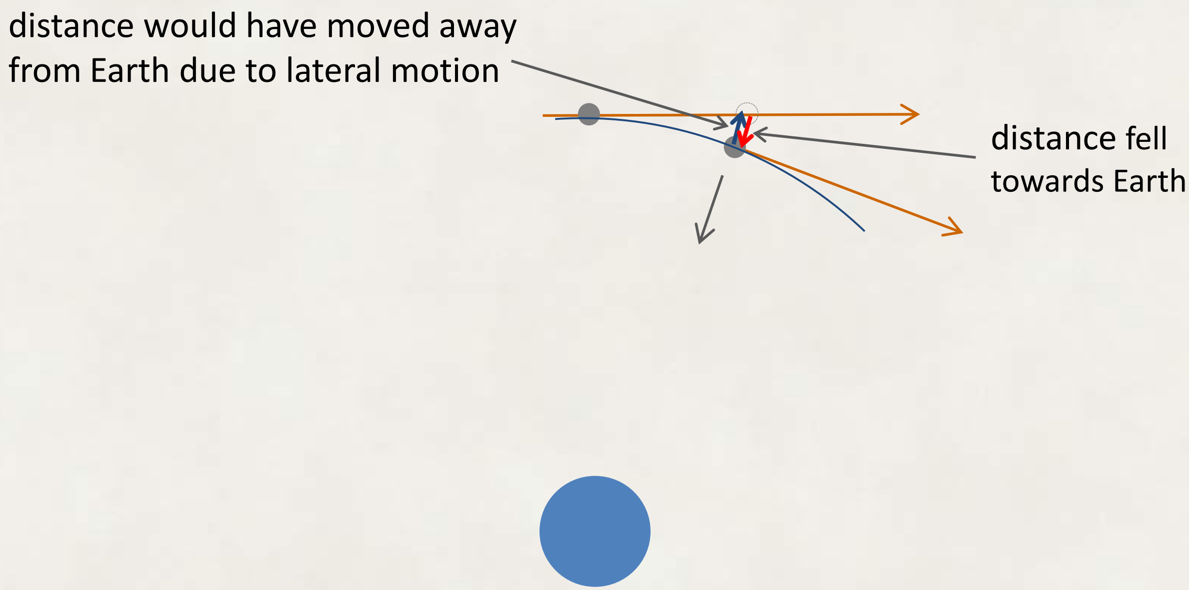


- Newton suggested an ingenious thought experiment to explain orbital motion.
- Suppose somebody is firing cannonballs form the top of a mountain horizontally, gradually firing at faster and faster speeds.
- The cannonball will hit the ground at a distance depending on its initial speed.
- As the speed of the cannonball is increased, it will hit the ground at a location farther and farther away from the mountain A,B,C...
- At some speed it moves so fast that it will not hit the ground. It travels in a circular path and return to the firing point, and continue circling the Earth. (that speed is about 8 km/s)
- If the speed is increased further, it will go into an elliptical orbit.

- If the speed is increased even further it will follow a parabolic or hyperbolic orbit and move away (escape) from the Earth.
- This limiting speed that a projectile will follow a parabolic orbit and escape is called the **escape velocity**. It is about 11 k/s from the Surface of the Earth.
  - **Escape velocity:** Speed needed to overcome the Earth's gravity and leave Earth.

Newton's cannonball experiment - Orbits





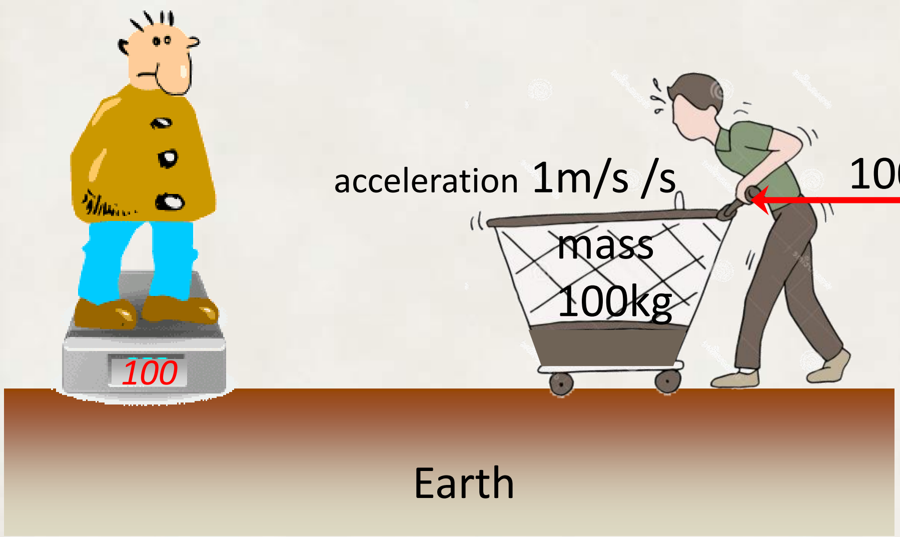
When orbiting the Earth, the distance cannonball falls toward the Earth (red arrow) is same as the distance it had moved away if there were no gravitational force and ball moved in a straight line sideways(blue arrow)

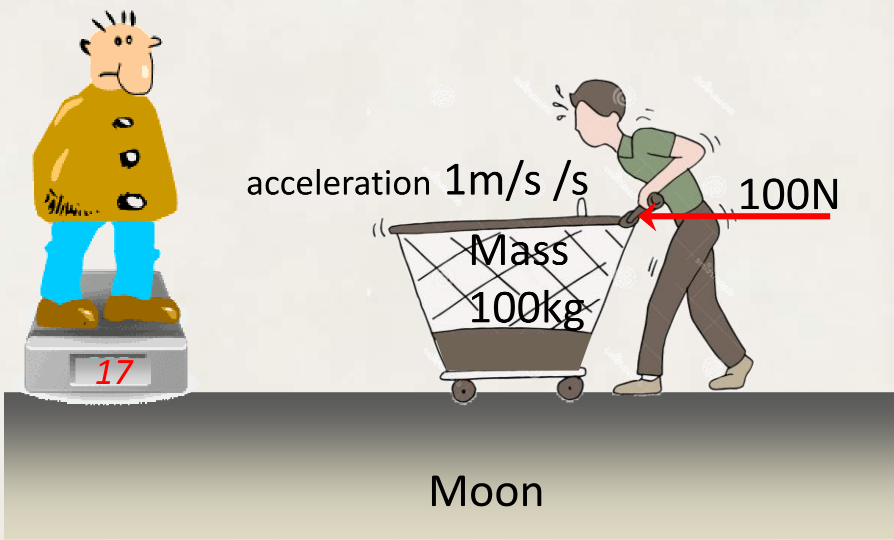
- In all those cases cannonball is always falling to (moving towards) the Earth, whether it hit the ground or not!
- So all objects orbiting the Earth are perpetually falling to the Earth. But the lateral orbital motion keeps them from ever actually hitting!
  - In the same way the Earth and planets are in a perpetual free fall to the Sun under Sun's gravity.

Weight and Mass

- **Mass:** is a measure of the amount of matter in an object.
- **Weight:** is the amount of gravitational pull on an object.
    - It is a force so has to measure in Newton

- Usually terms mass and weight are used loosely.
- People often use "weight" to mean "mass", and vice versa. It is technically wrong.
    - On the Earth gravity makes a 1 kilogram mass to exert about 9.8 Newton force.
    - That is what most scales measure. ie weight.
    - What scales really show is an estimate of mass (in kilograms or pounds) based on the weight of the object.
    - As long as we only work with objects on the Earth that works most of the time. But it cannot be used universally.





- Weight depends on the gravitational strength at the location.
  - On Earth weight of an object is the gravitational pull by the Earth (say 100kg)
  - On the Moon it would weight less because gravitation pull on the Moon is weaker (1/6 of Earth’s)
    - so a scale calibrated to read weight in kg on Earth would read about 17 kg on the Moon.
  - On Jupiter it would weight more (about 250 kg) because gravitational pull on Jupiter is stronger.
- In all above locations mass of the object is the same, 100kg.
  - You have to apply a force of same strength to accelerate (horizontally) at the same rate on the Moon as well as on the Earth.

H. G. Wells tells a story in which a very fat man wanted to rid himself of his fatness. The person who tells the story was the possessor of the recipe of a miraculous brew which could rid people of excessive weight. The fat man made the brew according to the recipe and drank it. And this is what happened.

"For a long time the door didn't open.

"I heard the key turn. Then Pyecraft's voice said, 'Come in.'

"I turned the handle and opened the door. Naturally I expected to see Pyecraft.

"Well, you know, he wasn't there!

"I never had such a shock in my life. There was his sitting-room in a state of untidy disorder, plates and dishes among the books and writing things, and several chairs overturned, but Pyecraft—

"'It's all right, o'man; shut the door,' he said, and then I discovered him.

"There he was right up close to the cornice in the corner by the door, as though someone had glued him to the ceiling. His face was anxious and angry. He panted and gesticulated. 'Shut the door,' he said. 'If that woman gets hold of it—'

Fig. 71. "There he was right up close to the cornice"

"I shut the door, and went and stood away from him and stared.

"'If anything gives way and you tumble down,' I said, 'you'll break your neck, Pyecraft.'

"'I wish I could,' he wheezed.

"'A man of your age and weight getting up to kiddish gymnastics—'

"'Don't,' he said, and looked agonised.

"'I'll tell you,' he said, and gesticulated.

"'How the deuce,' said I, 'are you holding on up there?'

"And then abruptly I realised that he was not holding on at all, that he was floating up there—just as a gas-filled bladder might have floated in the same position. He began a struggle to thrust himself away from the ceiling and to clamber down the wall to me. 'It's that prescription,' he panted, as he did so. 'Your great-gran—'

"He took hold of a framed engraving rather carelessly as he spoke and it gave way, and he flew back to the ceiling again, while the picture smashed on the sofa. Bump he went against the ceiling, and I knew then why he was all over white on the more salient curves and angles of his person. He tried again more carefully, coming down by way of the mantel.

"It was really a most extraordinary spectacle, that great, fat, apoplectic-looking man upside down and trying to get from the ceiling to the floor. 'That prescription,' he said. 'Too successful.'

"'How?'

"'Loss of weight—almost complete.'

"And then, of course, I understood.

"'By Jove, Pyecraft,' said I, 'what you wanted was a cure for *fatness*! But you always called it *weight*. You would call it weight.'

"Somehow I was extremely delighted. I quite liked Pyecraft for the time. 'Let me help you!' I said, and took his hand and pulled him down. He kicked about, trying to get foothold somewhere. It was very like holding a flag on a windy day.

"'That table,' he said pointing, 'is solid mahogany and very heavy. If you can put me under that—'

"I did, and there he wallowed about like a captive balloon, while I stood on his hearthrug and talked to him.

" ...'There's one thing pretty evident,' I said, 'that you mustn't do. If you go out of doors you'll go up and up....'

" ...I suggested he should adapt himself to his new conditions. So we came to the really sensible part of the business. I suggested that it would not be difficult for him to learn to walk about on the ceiling with his hands—

"'I can't sleep,' he said.

"But that was no great difficulty. It was quite possible, I pointed out, to make a shake-up under a wire mattress, fasten the under things on with tapes, and have a blanket, sheet, and coverlet to button at the side. He would have to confide in his housekeeper, I said; and after some squabbling he agreed to that. (Afterwards it was quite delight-

from *The truth about Pyecraft* by H. G. Wells

Measuring actual mass

- Measuring the actual mass (inertial mass) is more challenging.
- Inertial property of an object can be used measure its mass:
  - i.e. higher the mass (inertia) a larger force is needed to change its motion.
  - Newton's 2nd law: $F=ma$

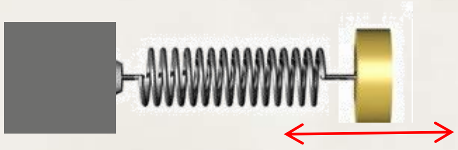
    force = mass x acceleration

Apply a known force and measure the acceleration to estimate the mass of an object

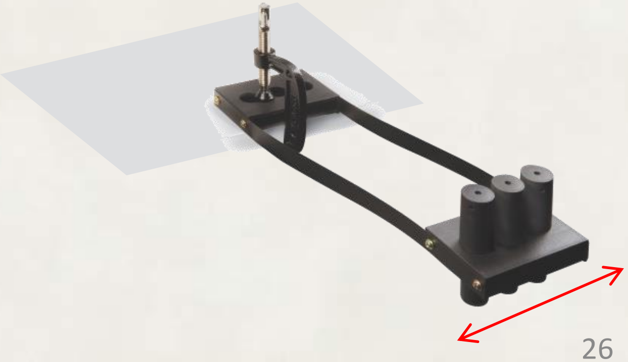
www.airspacemag.com/daily-planet/how-do-astronauts-weigh-themselves-space-180953884/

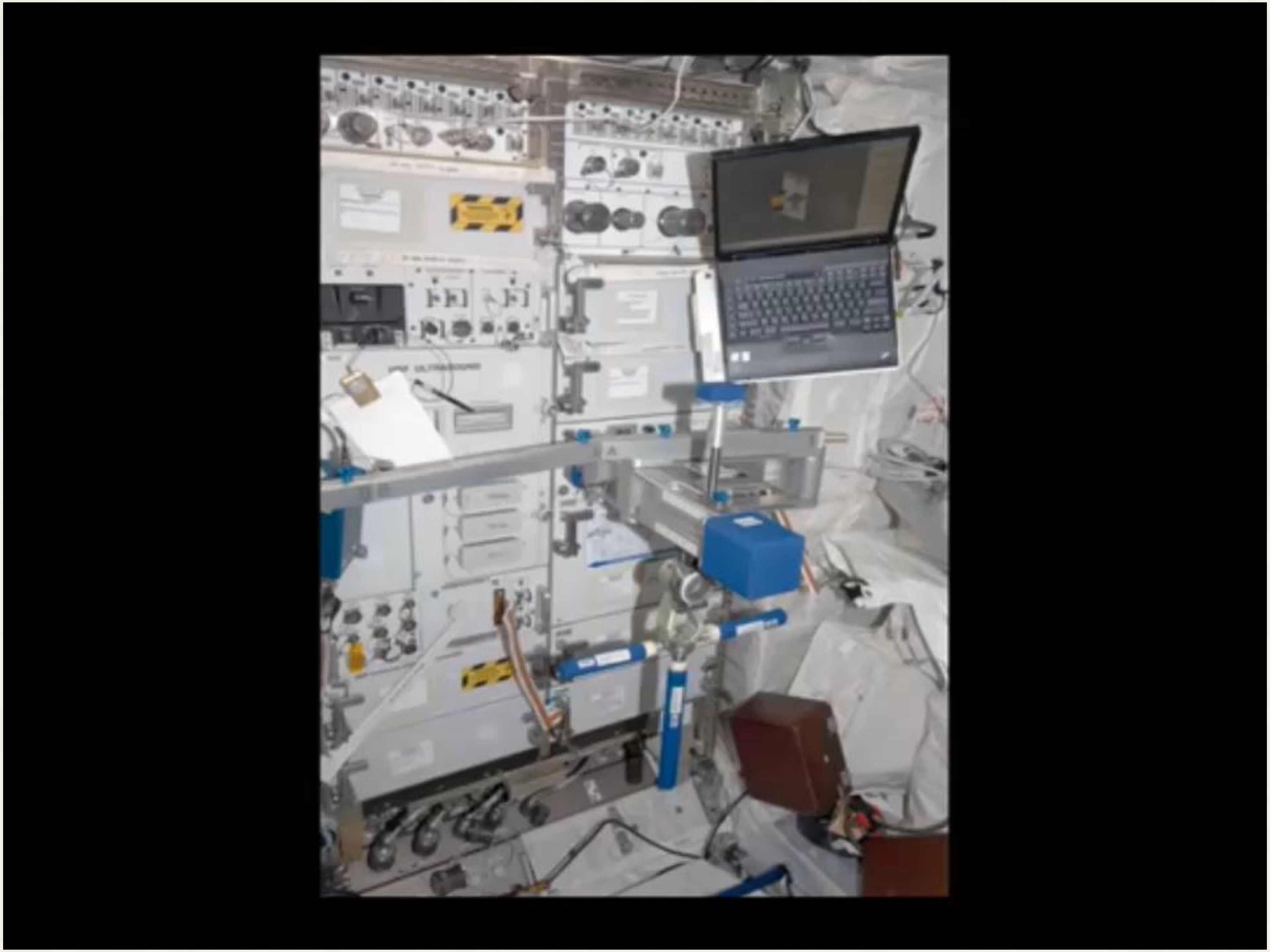
Using an inertia balance at the international space station

Inertia balance: Object moves back and forth (oscillates) due to the restoring force of a spring.



- More massive objects accelerates/moves slower and takes a longer time to complete an oscillation.
- That can be used to estimate the mass of an object regardless of the gravity at the location.



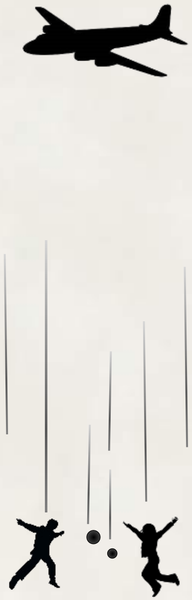


Measuring mass at the International space station using a linear Accelerometer   www.youtube.com/watch?v=qE4OoE93fX0

www.youtube.com/watch?v=8rt3udip7l4

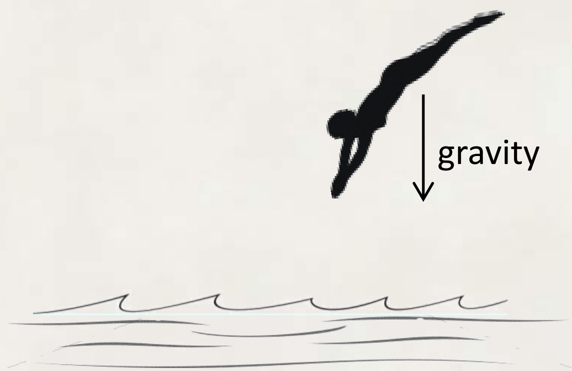
Measuring mass at the International Space Station using an inertia balance

Free falls and weightlessness

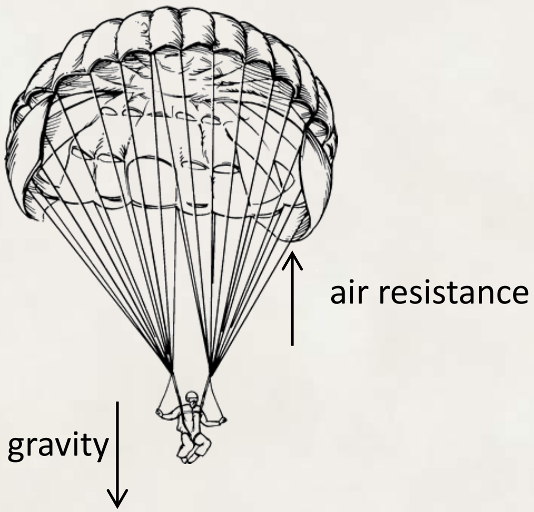


Suppose few objects were dropped from an airplane and falling freely (ignore the air resistance for now).

- Everything falls at the same rate (acceleration) under gravity.
- Therefore they will stay together while falling.
- For somebody falling with them, they would appear as floating right next to him all the time.
- No other support or force is needed keep them together.
  - as if they are weightless.



Jumping from a diving board is a free fall and one could experience weightlessness.



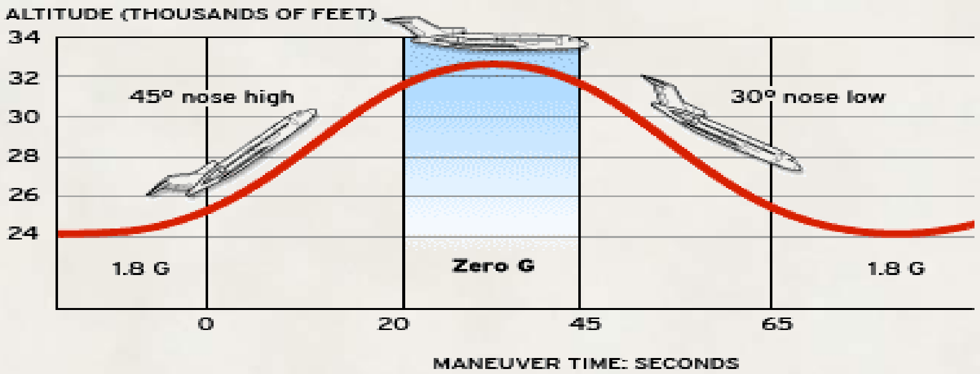
Jumping with an open parachute is not a free fall.

Weightlessness in outer space

Experiencing zero gravity aboard the International space station

- For the same reason inside an spacecraft in outer space things appear to be floating and experience no gravity.
  - But 300-400 km above, Earth's gravity is not very different from what it is on the ground (about 90%)
- What really happens is that the spacecraft is in a perpetual free fall to Earth (or whatever gravity it is subjected to) with everything in it.
- Since everything is falling together, no support or force is needed to keep them in place.
- Just like we do not feel the gravitational pull of the Sun since Earth is always falling to the Sun.

Weightlessness



Zero gravity flight maneuver

science.nasa.gov/science-news/science-at-nasa/fluid/zero-gplane/

An airplane undergoing a free fall has the same weightlessness as an orbiting satellite for a short duration (Zero-g Cop. https://www.gozerog.com)

- Same can be experienced inside an airplane if allowed to fall freely.

Zero Gravity ride https://www.youtube.com/watch?v=PosRfeUoPHM

Review Questions

- Who was the fist to show that natural state of movement of an object is uniform speed in a straight line?
- Why don’t we see objects keep moving in straight lines as required by Newton’s first law of motion on Earth?
- Why do passengers in a car jerk forward when the car suddenly stops?
- Why do you feel you pushed sideways when a vehicle making a sharp turn at high speed?
- Why do airplanes tilt sideways when they turn?
- According to Galileo what is the nature of the motion of a falling body?
- Suppose you are swing a threaded ball over your head and suddenly release it. In which direction would it move after you release it?
- Why does a gun recoil when it is fired?
- Can one experience weightlessness while jumping with a deployed parachute? Why?
- What would be the weight of an object at the center of the Earth (suppose you drill a hole to get there and measure the weight)