PHYS 401 Electromagnetic Theory, Homework #2. Due on Friday 9/14

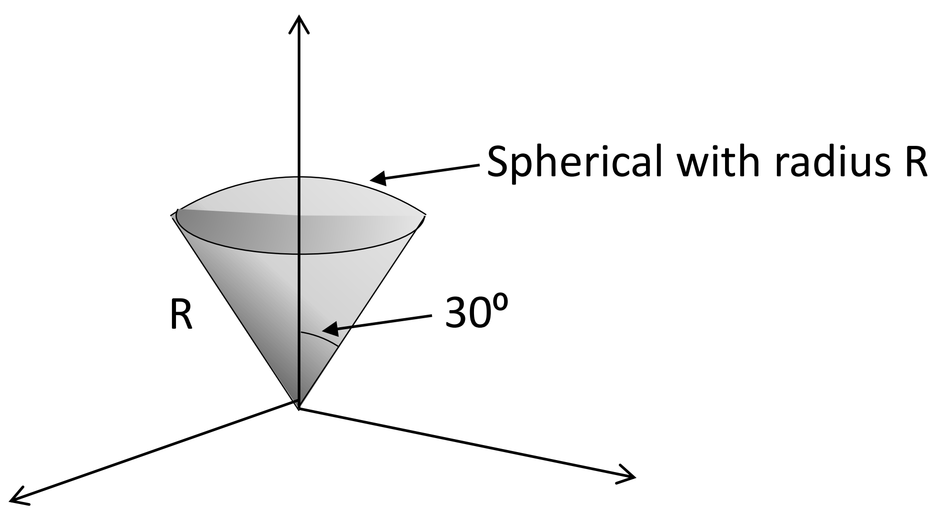
1) Show that the divergence of curl is always zero. Then check that for the function $x^2\hat{x} + 3xz^2\hat{y} - 2\mathrm{xz}\hat{z}$.

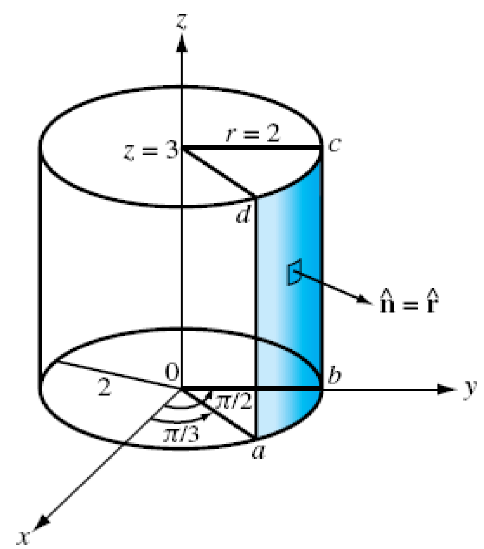
2) A vector field $\vec{V} = r^3\hat{r}$ is defined in the region between two concentric cylindrical surfaces with r = 1 and r = 2. Both cylinders are extending from z = 0 to z = 5. Verify the divergence theorem by evaluating

a) $\oint_{surface} V \cdot da$

b) $\oint_{volume} \nabla \cdot V dv$

3) A vector field is given in a cylindrical coordinate system by $\vec{B} = \frac{\hat{z}\cos\phi}{r}$. Verify Stokes's theorem for a segment of a cylindrical surface defined by $r = 2$, $\pi/3 \le \varphi \le \pi/2$, $0 \le z \le 3$ as shown in the diagram below.





4) A vector field $\vec{V}$ in spherical coordinates is given by:

$$\vec{V} = r^2 sin\theta\hat{r} + r^2 scos\theta\hat{\theta} + r^2 tan\theta\hat{\phi} \text{ ‘}$$

Prove the divergence theorem for this field for a conical volume with spherical top as shown in the above figure (left)

5) An electric charge Q is uniform ally smeared on the surface of a spherical shell of radius R. Write an expression in a suitable coordinate system to represent the volume charge density ρ of that charge distribution. Make sure that the volume integral of ρ is equal to Q.

6) Evaluate the following integrals:

$i)\ \int_0^5 (2x^2 - 3x + 1)\delta(x+1)dx$

$ii)\ \int_0^5 (4x^3 - 5x + 2)\delta(x-2)dx$

$iii)$ if the function $f(x) = 0$ when $x = x_0$ (and only when $x = x_0$)

$$\text{show that } \delta(\mathrm{f(x)}) = \frac{\delta(x - x_0)}{\left|\frac{df}{dx}\right|_{x=x_0}}$$

$iv)$ show that $x\frac{d}{dx}\delta(x) = -\delta(x)$ (Hint: use integration by parts)

$v)$ use the result (iii) to evaluate $\int_{-\infty}^{\infty} x^2\delta(\sin x)dx$

(note: $\sin x = 0$ for more than one value of x)