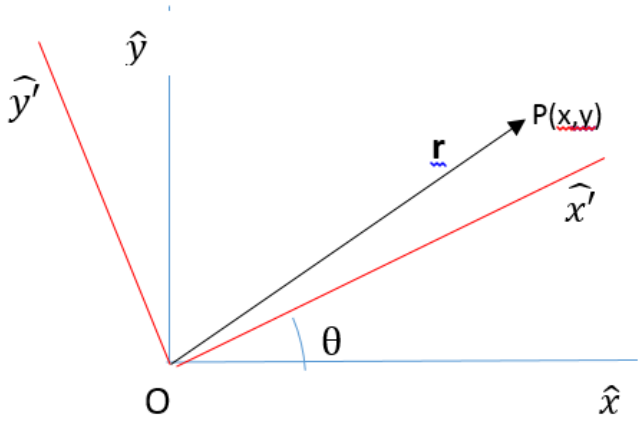
PHYS 401 Homework #1 , Due on 8/29

1) $\vec{a}, \vec{b}$ are the positions vectors of points A, B. Point C is on the line AB divides AB in m:n ratio. Show that the position vector of C is given by : $\vec{c} = \frac{n\vec{a}+m\vec{b}}{m+n}$.

2) Position vectors of points A,,B,C in an orthonormal Cartesian coordinate system is given by (2,0,5), (-2, 4, 5) and (1,1,4) respectively. Show that they are collinear.

3) Position vectors of four points are in a Cartesian coordinate system are (1,0,3) , (5,2,1), (-1,3,4) and (4,2,3). Show that they are coplanar.



4) As shown in the above diagram, coordinates of the point P in XY Cartesian coordinate system is (x,y).

I. Write the position vector of P in terms of unit vectors $\hat{x}, \hat{y}$.

II. Another Cartesian coordinate system has the same origin, but its axes are at an angle θ to the other coordinate system. Write unit vectors $\hat{x}', \hat{y}'$ of that system in terms of $\hat{x}, \hat{y}$.

III. Write the position vector of P in terms of unit vectors $\hat{x}', \hat{y}'$

IV. What are the coordinates of P in the new coordinate system.

5) $\vec{a}, \vec{b}$, $\vec{c}, \vec{d}$ are the position vectors of points A, B,C and D. In which direction the vector $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$ is pointing (explain your reasoning).

Show that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [(\vec{a} \times \vec{b}) \cdot \vec{d}]\vec{c} - [(\vec{a} \times \vec{b}) \cdot \vec{c}]\vec{d}$

$= [(\vec{c} \times \vec{d}) \cdot \vec{a}]\vec{b} - [(\vec{c} \times \vec{d}) \cdot \vec{b}]\vec{a}$

Use the above result to show how a given vector can be uniquely written in terms of components along three non-coplanar vectors.

Extra Credit:

In a building plan two long straight pipes are specified using Cartesian coordinates as follows:

Pipe A: diameter 0.8, axis through points (2,5,3) and (7, 10, 8),

Pipe B: diameter 1.0, axis through points (0,6,3) and (-12,0,9)

Are the pipes well separated to avoid intersection. (hint: calculate the shortest distance between two axis)