6 Magnetization

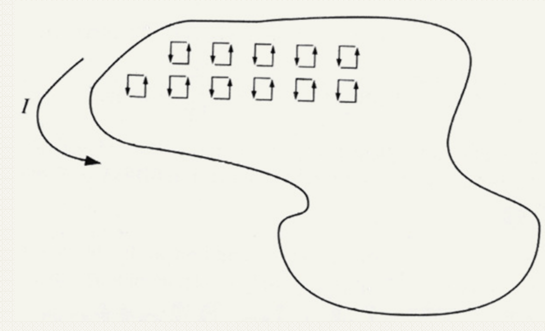
Diamagnets, Paramagnets, Ferromagnets

- Magnetism in a material is a result of tiny dipole moments in atomic scale.
- Usually they each other due to their random orientation.
- In an external magnetic field those dipoles align and the medium becomes polarized (or magnetized)

- Paramagnets: magnetization is parallel to the applied magnetic field
- Diamagnets: magnetization is opposite to the applied magnetic field
- Ferromagnets: retain a substantial magnetization indefinitely after the external field has been removed.

Torques and Forces on Magnetic Dipoles

A magnetic dipole experiences a torque in a magnetic field :

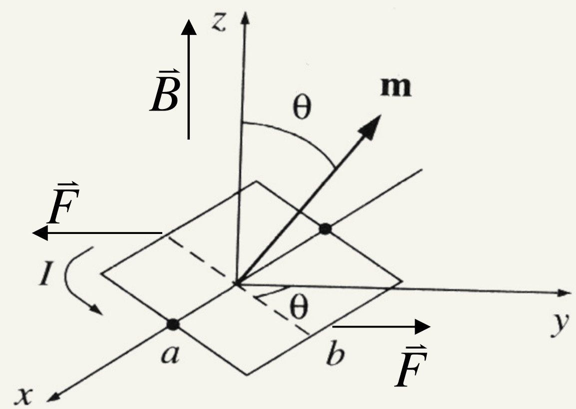


Torque on a rectangular current loop in a uniform magnetic field

$$\vec{N} = aF\sin\theta\,\hat{i}$$

$$= a(bIB)\sin\theta\,\hat{i}$$

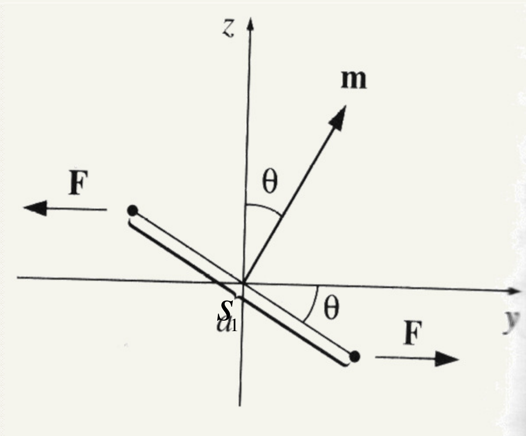
$$= mB\sin\theta\,\hat{i} \qquad m = abI$$



$$\boxed{\vec{N} = \vec{m}\times\vec{B}}$$

magnetic dipole moment

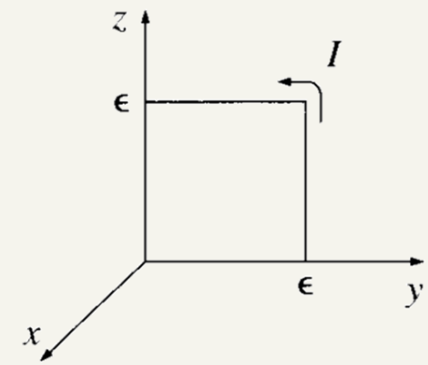
•This torque lines the magnetic dipole up parallel to $\vec{B}$: paramagnetism.



(b)

In a uniform magnetic field, the net force is zero.

$$\vec{F} = I\oint(d\vec{l}\times\vec{B}) = I(\oint d\vec{l})\times\vec{B} = 0$$



But in a nonuniform magnetic field, there is a net force

For an infinitesimal loop with dipole moment m in a field B

$$\vec{F} = I\oint(d\vec{l}\times\vec{B}(r)) = I\int\Big[dy\hat{y}\times\vec{B}(0,y,0) + dz\hat{z}\times\vec{B}(0,\varepsilon,z) - dy\hat{y}\times\vec{B}(0,y,\varepsilon) - dz\hat{z}\times\vec{B}(0,0,z)\Big]$$

$$= I\int\Big\{dy\hat{y}\times\Big[\vec{B}(0,y,0)-\vec{B}(0,y,\varepsilon)\Big] + dz\hat{z}\times\Big[\vec{B}(0,\varepsilon,z)-\vec{B}(0,0,z)\Big]\Big\}$$

$$= I\int\left(-dy\hat{y}\times\varepsilon\left.\frac{\partial\vec{B}}{\partial z}\right|_{(0,y,0)} + dz\hat{z}\times\varepsilon\left.\frac{\partial\vec{B}}{\partial y}\right|_{(0,0,z)}\right)$$

$$= I\varepsilon^2\left(\hat{z}\frac{\partial\vec{B}}{\partial y} - \hat{y}\frac{\partial\vec{B}}{\partial z}\right) = m\left(\hat{y}\frac{\partial B_x}{\partial y} - \hat{x}\frac{\partial B_y}{\partial y} - \hat{x}\frac{\partial B_z}{\partial z} - \hat{z}\frac{\partial B_x}{\partial z}\right)$$

$$= m\left(\hat{y}\frac{\partial B_x}{\partial y} - \hat{x}\frac{\partial B_y}{\partial y} - \hat{x}\frac{\partial B_z}{\partial z} + \hat{z}\frac{\partial B_x}{\partial z}\right) = m\left[\hat{y}\frac{\partial B_x}{\partial y} - \hat{x}\left(\frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}\right) + \hat{z}\frac{\partial B_x}{\partial z}\right]$$

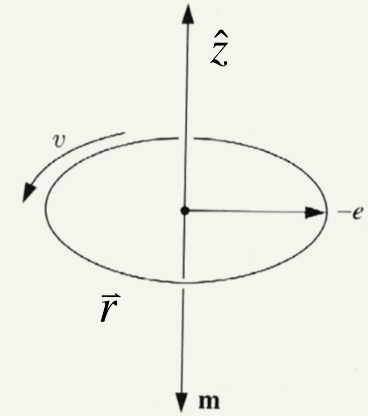
$$= m\left[\hat{x}\frac{\partial B_x}{\partial x} + \hat{y}\frac{\partial B_x}{\partial y} + \hat{z}\frac{\partial B_x}{\partial z}\right] \quad \left(\because \nabla.\vec{B} = 0\right)$$

$$= m\nabla B_x = m\nabla(\hat{x}.\vec{B}) = \nabla(m\hat{x}.\vec{B}) = \nabla(\hat{m}.\vec{B})$$

$$\vec{F} = \nabla(\vec{m}\cdot\vec{B})$$

Effect of a Magnetic Field on Atomic Orbits

A classical model:



Assume the orbit of an electron a circle of radius r

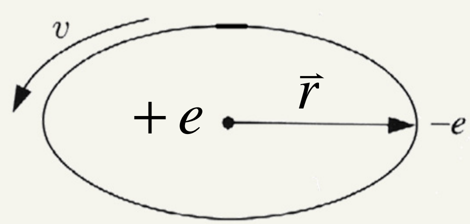
since its period $T = \dfrac{2\pi r}{v} = \dfrac{2\pi}{w_c} << 1$

Its motion gives a steady-like current $I = \dfrac{e}{T} = \dfrac{ev}{2\pi r}$

With a dipole moment $\vec{m} = \pi r^2 I = -\dfrac{1}{2} evr\hat{z}$

Which is subjected to force in a magnetic field $\vec{B}$ $= \vec{m} \times \vec{B}$

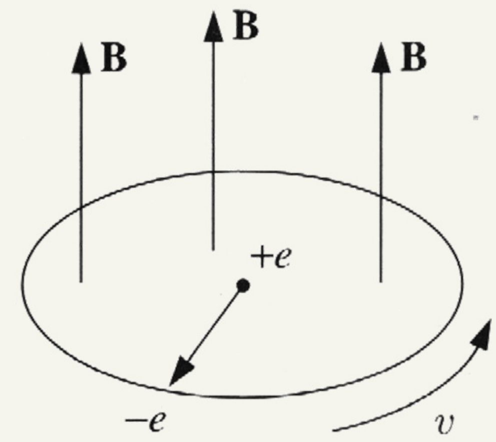
Without a magnetic field



$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = m_e \frac{v^2}{r}$$

In a magnetic field $\vec{B}$

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} + ev'B = m_e\frac{v'^2}{r}$$



$$ev'B = \frac{m_e}{r}(v'^2 - v^2)$$ (assume r = constant)

$$= \frac{m_e}{r}(v'+v)(v'-v) \quad \Rightarrow \Delta v = \frac{erB}{2m_e}$$

$$(\Delta v << v \quad v + v' \approx 2v')$$

Since magnetic moment $\vec{m} = -\frac{1}{2}evr\hat{z}$

The B field changes the dipole moment $\Delta\vec{m} = -\frac{1}{2}e(\Delta v)r\hat{k} = -\frac{e^2r^2}{4m_e}\vec{B}$

This change in the dipole moment is opposite to the direction of B.

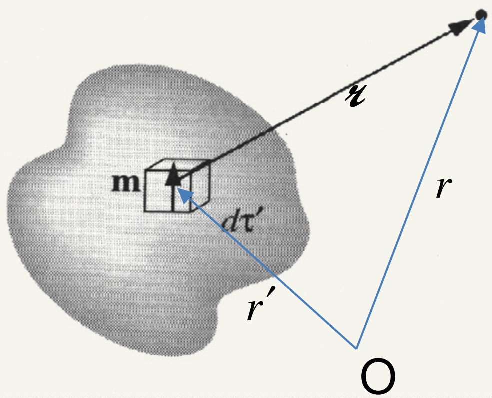
Since usually atoms (and electron orbits)are randomly oriented, net magnetization is zero. But in a magnetic field each atom picks up an extra dipole moment in opposite direction to field. Thus resulting a diamagnetism.

Diamagnetism and paramagnetism are usually very weak, can be neglected in most cases.

6.2 The Field of a Magnetized Object **Bound Currents**

magnetization $\vec{M} \equiv magnetic\ dipole\ moment\ \ per\ unit\ volume$

for a single dipole $\vec{m}$



$$\vec{A} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{\mathscr{r}}}{\mathscr{r}^2} \qquad \vec{m} = \vec{M}d\tau', \qquad \mathscr{r} = r - r'$$

$$\text{total } \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int\frac{\vec{M}\times\hat{\mathscr{r}}}{\mathscr{r}^2}d\tau'$$

$$= \frac{\mu_0}{4\pi}\int(\vec{M}\times\nabla'\frac{1}{\mathscr{r}})d\tau' \qquad \left(\because (\nabla'\frac{1}{\mathscr{r}} = \nabla'\frac{1}{|\vec{r}-\vec{r}'|} = \nabla'\frac{1}{|\vec{r}'-\vec{r}|} = -\frac{\widehat{(r'-r)}}{\mathscr{r}^2} = \frac{\widehat{r-r'}}{\mathscr{r}^2} = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2})\right)$$

$$= \frac{\mu_0}{4\pi}[\int\frac{1}{\mathscr{r}}(\nabla'\times\vec{M})d\tau' - \int\nabla'\times(\frac{1}{\mathscr{r}}\vec{M})d\tau'] \quad \left(\because \nabla'\times(\frac{1}{\mathscr{r}}\vec{M}) = \frac{1}{\mathscr{r}}(\nabla'\times\vec{M}) - \vec{M}\times(\nabla'\frac{1}{\mathscr{r}})\right)$$

$$= \frac{\mu_0}{4\pi}\int\frac{1}{\mathscr{r}}(\nabla'\times\vec{M})d\tau' + \frac{\mu_0}{4\pi}\oint\frac{1}{\mathscr{r}}(\vec{M}\times d\vec{a}') \quad \left(\because \int\nabla\times\vec{A}d\tau = \oint\vec{A}\times d\vec{a} \ \ \text{problem 1.61b}\right)$$

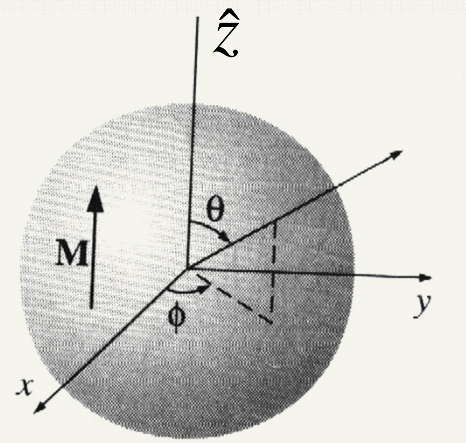
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{1}{\mathscr{r}}(\nabla' \times \vec{M})d\tau' + \frac{\mu_0}{4\pi}\oint \frac{1}{\mathscr{r}}(\vec{M} \times d\vec{a}')$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}_b(r')}{\mathscr{r}} d\tau' + \frac{\mu_0}{4\pi}\int_S \frac{\vec{K}_b(r')}{\mathscr{r}} d\vec{a}'$$

Potential (and field) of a magnetized object is the same as would be produced by a bound volume current $J_b(r') = \nabla' \times \vec{M}(r')$ and a surface current $K_b(r') = \vec{M}(r') \times \hat{n}$

cf: in ES volume charge $\rho_b = -\nabla \cdot P$ surface charge $\sigma_b = \vec{P} \cdot \hat{n}$

Example: Magnetic field of a uniformly magnetized sphere



Since $\vec{M}$ is uniform:

$$\vec{J}_b = \nabla \times \vec{M} = 0 \quad \vec{K}_b = \vec{M} \times \hat{n} = M \sin\theta\,\hat{\varphi}$$

This is similar to the currents on a uniformly charged rotating spherical shell (Ch. 5)

$$\vec{K} = \sigma\vec{v} = \sigma\omega R \sin\theta\,\hat{\varphi}$$

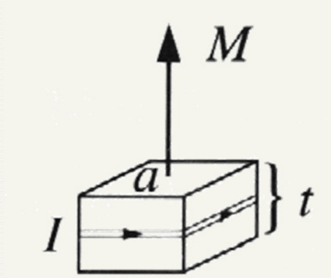
$$\text{field produced } \vec{B} = \frac{2}{3}\mu_0 \sigma z R \omega \hat{z} \qquad (inside)$$

$$inside \qquad \vec{B} = \frac{2}{3}\mu_0 \vec{M} \qquad (uniform)$$

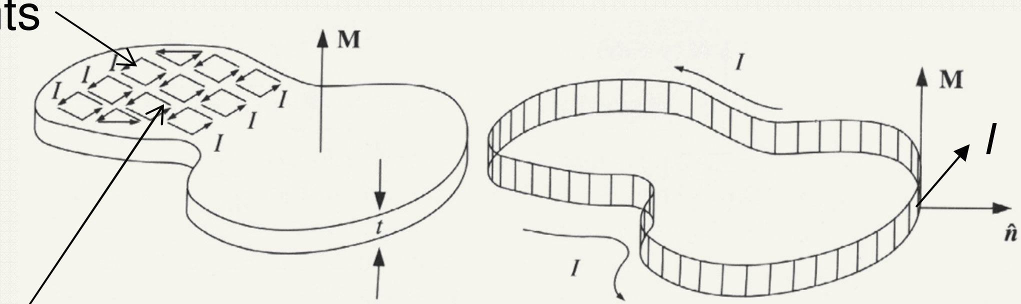
$$\text{outside field is same as that of a dipole} \qquad \vec{m} = \frac{4}{3}\pi R^3 \vec{M}$$

Physical Interpretation of Bound Currents

For uniform $\vec{M}$



Dipole currents



Internal loop currents cancel

Only the current flowing around the boundary remains

$$\vec{m} = \vec{M}at$$

$$= Ia \qquad \Rightarrow I = Mt$$

$$= K_b t \;\Rightarrow K_b = M$$

$$\boxed{\vec{K}_b = \vec{M} \times \hat{n}}$$

Each charge moves in a small loop, but the net effect is a macroscopic current flowing over the surface.

For non-uniform $\vec{M}$ internal currents do not cancel.

A nonuniform M in y contributes to a current in x direction.

$$I_x = [M_z(y+dy) - M_z(y)]dz$$

$$= \frac{\partial M_z}{\partial y} dydz$$

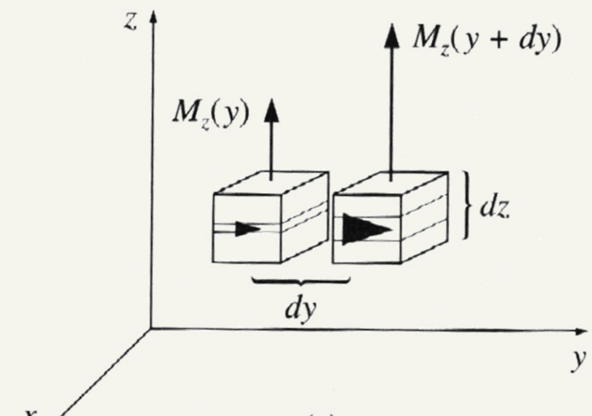
Current density: $J_x = \frac{I_x}{dydz} = \frac{\partial M_z}{\partial y}$

A nonuniform M in z, also contributes to current in x direction

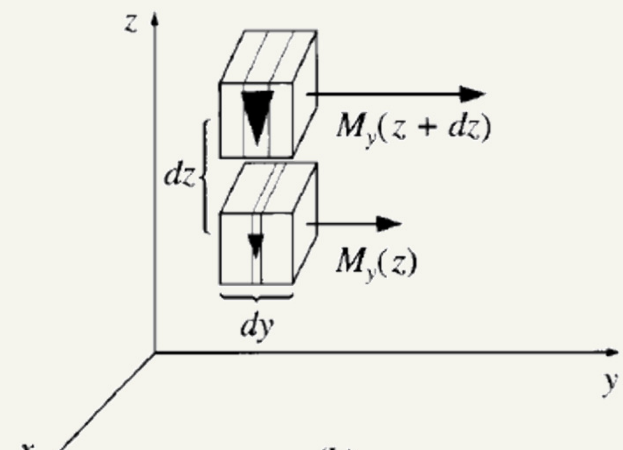
$$J_x = -\frac{\partial M_y}{\partial z}$$

$$\therefore (J_b)_x = \frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z}$$

$$\Rightarrow \boxed{\vec{J}_b = \nabla \times \vec{M}} \qquad \nabla \cdot J_b = 0$$



(a)



(b)

6.3 The Auxiliary Field $\vec{H}$

In Electrostatics $\rho_b = -\nabla \cdot \vec{P}$ $\qquad \nabla \cdot \vec{E} = \dfrac{\rho_b + \rho_f}{\varepsilon_0}$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \qquad \nabla \cdot \vec{D} = \rho_f$$

In a magnetized medium there are bound currents $J_b = \nabla \times \vec{M}$

So the total current is sum of bound currents and other (free) currents

$$J = J_b + J_f$$

So the Ampere law: $\nabla \times \vec{B} = \mu_0 \vec{J} = \mu_0 (\vec{J}_b + \vec{J}_f) = \mu_0 (\nabla \times \vec{M} + \vec{J}_f)$

$$\Rightarrow \nabla \times \left(\frac{1}{\mu_0}\vec{B} - \vec{M}\right) = \vec{J}_f$$

$$\text{or } \nabla \times \vec{H} = \vec{J}_f \quad \text{where } \vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$$

In integral form $\oint \vec{H} \cdot d\vec{l} = I_{f,enc}$ Total free current passing through an Amperian loop

Problem 6.12 An infinitely long cylinder, of radius R, carries a "frozen-in" magnetization. parallel to the axis,

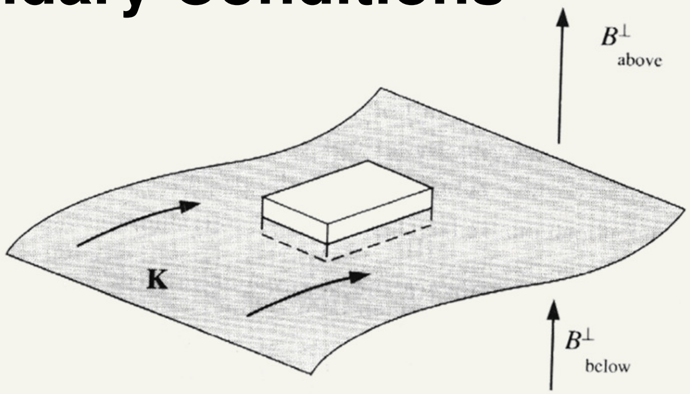
$$\mathbf{M} = ks\,\hat{\mathbf{z}},$$

where k is a constant and s is the distance from the axis; there is no free current anywhere. Find the magnetic field inside and outside the cylinder by two different methods:

(a) As in Sect. 6.2, locate all the bound currents, and calculate the field they produce.

(b) Use Ampère's law (in the form of Eq. 6.20) to find **H**, and then get **B** from Eq. 6.18. (Notice that the second method is much faster, and avoids any explicit reference to the bound currents.)

Boundary Conditions

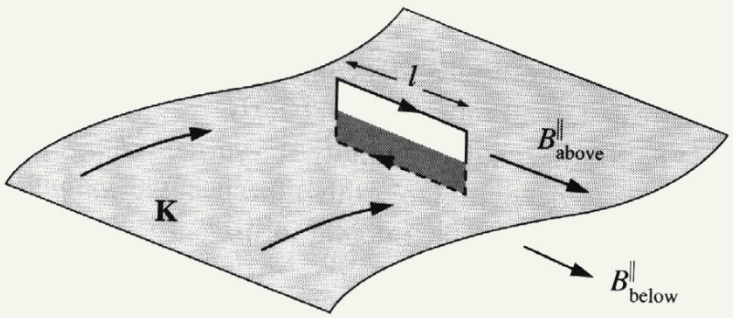


$$\oint \vec{B}\cdot d\vec{a} = 0 \Rightarrow B^{\perp}_{above} = B^{\perp}_{below}$$

Since $\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$

$$H^{\perp}_{above} + M^{\perp}_{above} = H^{\perp}_{below} + M^{\perp}_{below}$$

$$H^{\perp}_{above} - H^{\perp}_{below} = -(M^{\perp}_{above} - M^{\perp}_{below})$$



$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$\Rightarrow B^{\parallel}_{above} - B^{\parallel}_{below} = \mu_0 K$$

$$\oint \vec{H}\cdot d\vec{l} = I_{f,enc}$$

$$H^{\parallel}_{above} - H^{\parallel}_{below} = \mu_0 K_f$$

linear and Nonlinear Media

6.4. Magnetic Susceptibility and Permeability

In Electrostatics $\vec{P} = \varepsilon_0 \chi_e \vec{E}$ $(linear\ dielectric)$

Polarization ↑ electric susceptibility ↑

$$(\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}) \qquad \vec{D} = \varepsilon \vec{E} \qquad \varepsilon = \varepsilon_0 (1 + \chi_e)$$

Similar parameters can be defined for magnatostatics. In paramagnetic and diamagnetic materials magnetization is sustained by the applied magnetic field and proportional to B

As a convention magnetization is written in terms of H,

For a linear media $\vec{M} = \chi_m \vec{H}$ χ_m magnetic susceptibility

χ_m>0 paramagnets, χ_m <0 dimagnets. typically $\approx 10^{-5}$

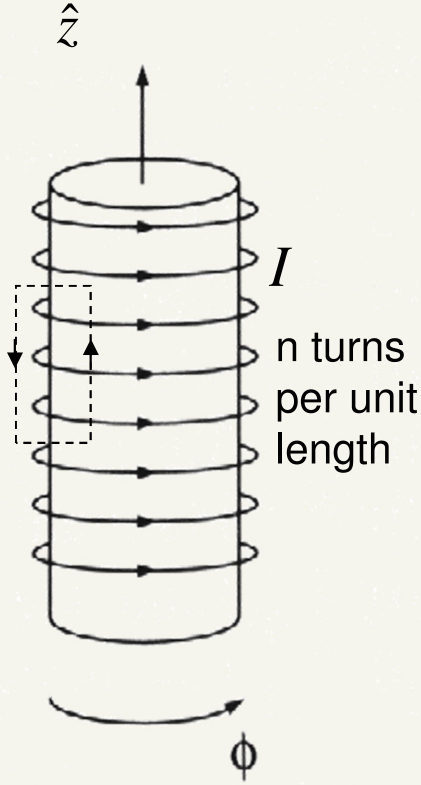
$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (1 + \chi_m) \vec{H}$$

$$= \mu \vec{H} \quad \text{where} \quad \mu = \mu_0 (1 + \chi_m)$$

μ permeability ; μ_0 permeability of free space

Example: Magnetic field of an infinite length solenoid filled with linear material of susceptibility χ_m

Since B depends on bound currents, apply ampere Law for H



$$\int H \cdot dl = I_{enc}$$

$$\vec{H} = nI\hat{z}$$

$$\vec{B} = \mu_0(1+\chi_m)nI\hat{k}$$

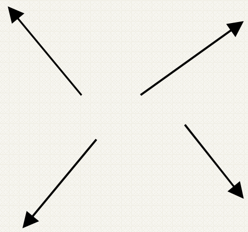
Bound surface currents $\vec{K}_b = \vec{M} \times \hat{n} = \chi_m nI(\hat{z}\times\hat{n}) = \chi_m nI\hat{\varphi}$

If the medium is paramagnetic (χ_m>0) and field is (slightly) enhanced and if the medium diamagnetic field is reduced.

6.4.2 Ferromagnetism

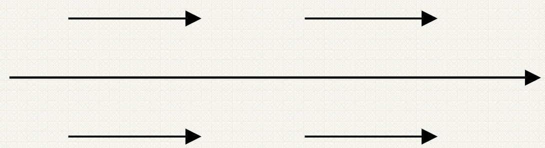
Paramagnetic

no B

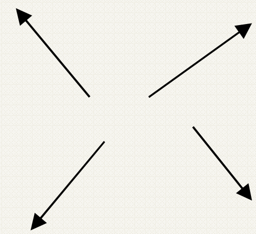


$\vec{M} = 0$

applied $\vec{B}$



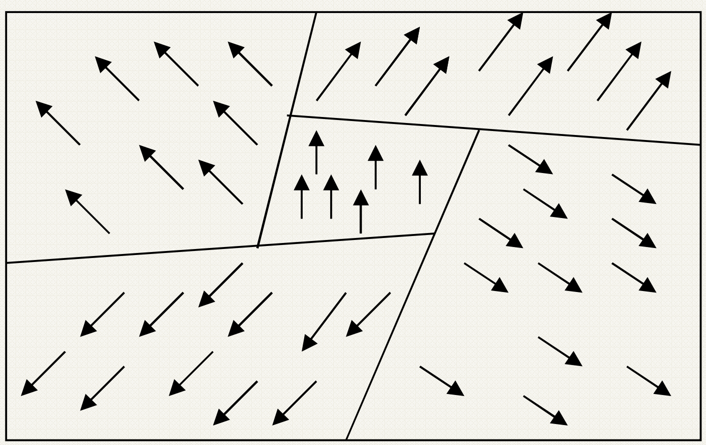
remove $\vec{B}$



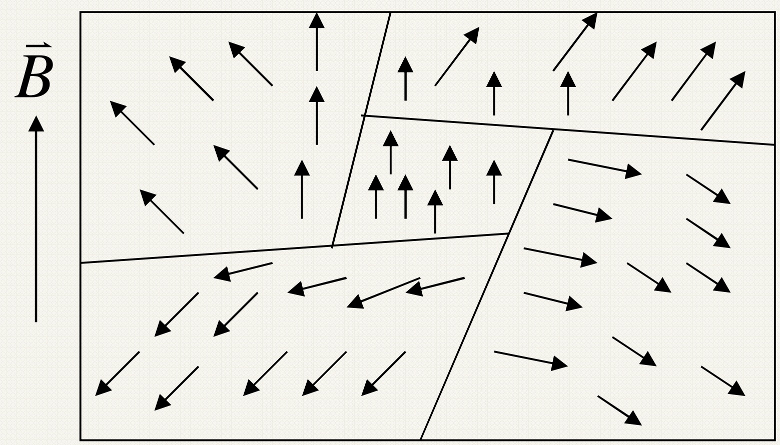
$\vec{M} = 0$

Ferromagnet

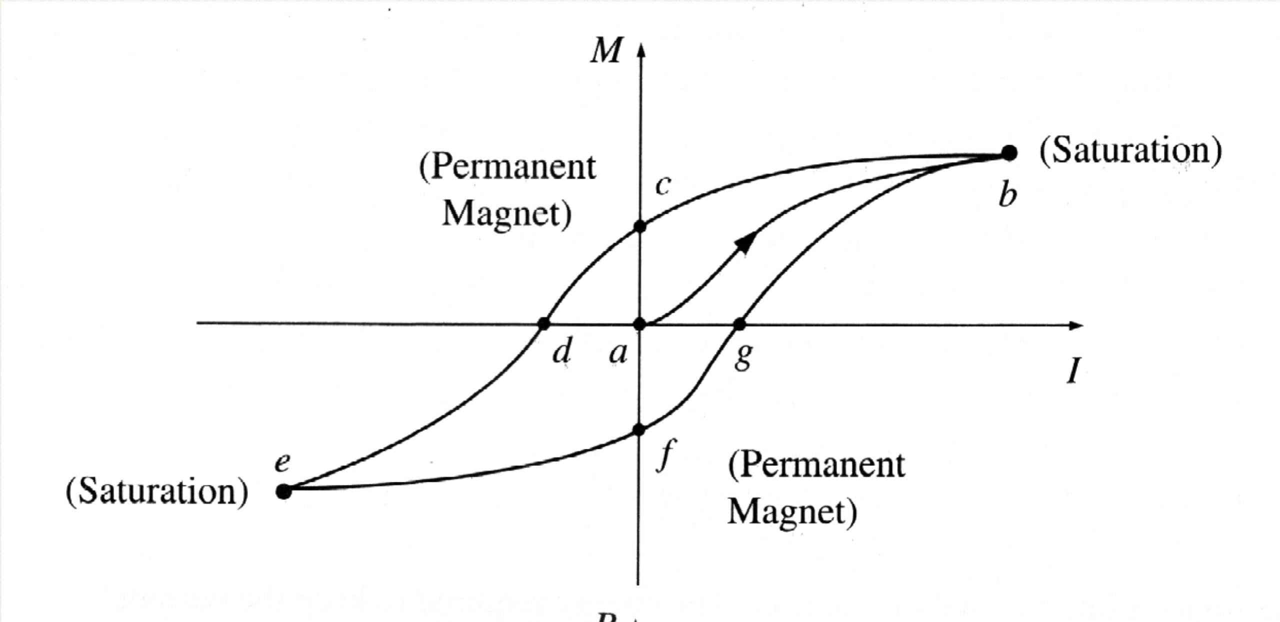
no $\vec{B}$

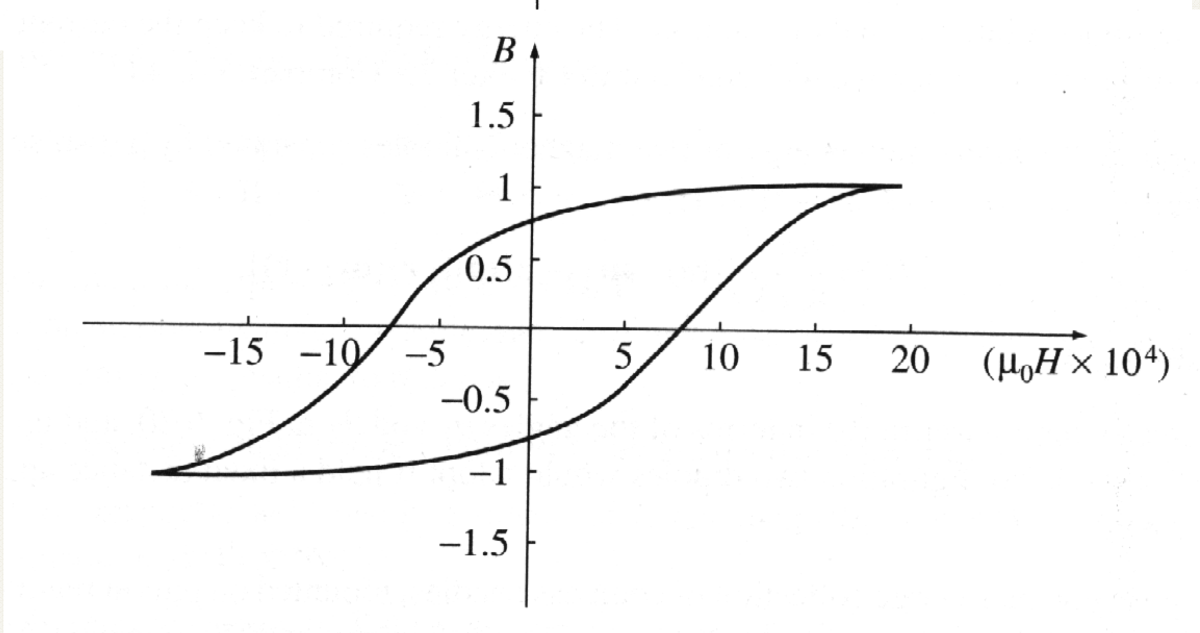


with $\vec{B}$



6.4.2(2)


M
(Permanent
Magnet)
c
(Saturation)
b
d
a
g
I
e
(Saturation)
f
(Permanent
Magnet)


B
1.5
1
0.5
−15
−10
−5
5
10
15
20
($\mu_0 H \times 10^4$)
−0.5
−1
−1.5

6.4.2(3)

Iron is ferromagnetic,

but if $T>T_{curie}$ (770 ° for iron)

← curie point

it become a paramagnetic.

This is a phase transition.