Chapter 4: Electrostatic Fields in Matter

4.1 Polarization

4.2 The Field of a Polarized Object

4.3 The Electric Displacement

4.4 Self-Consistance of Electric Field
and Polarization; Linear Dielectrics

4.1 Polarization

4.1.1 Dielectrics

4.1.2 Induced Dipoles and Polarizability

4.1.3 Alignment of polar molecules

4.1.4 Polarization and Susceptubility

4.1 Polarization

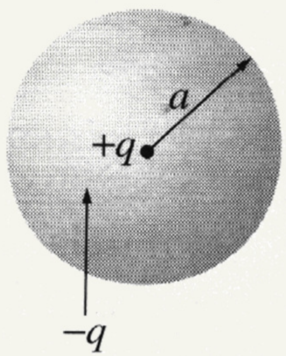
Dielectrics

According to their response to electrostatic field, most materials belong to two classes:

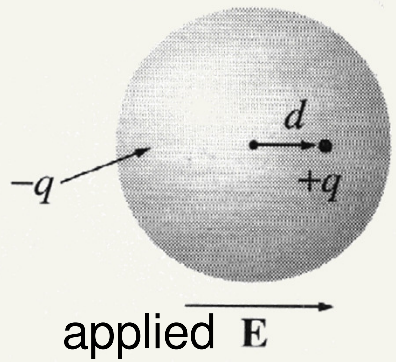
- conductors: charges free to move in the material
- insulators (or dielectrics): all charges are attached to specific atoms or molecules.

Stretching and rotating are the two principle mechanisms by which electric fields can distort the charge distribution of a dielectric atom or molecule.

4.1.2 Induced Dipoles and Polarizability



A neutral atom



Polarized atom

Induced dipole

$$p = qd$$

$$\vec{p} = \alpha \vec{E}$$

↑ Atomic polarizability

A simple estimate:

The field at a distance *d* from the center of a uniform charge sphere

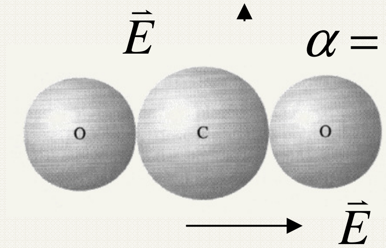
$$E_e = \frac{1}{4\pi\varepsilon_0}\frac{q_{in}}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{d^2}\frac{d^3}{a^3} = \frac{1}{4\pi\varepsilon_0}\frac{qd}{a^3}$$

At equilibrium,

$$E = E_e = \frac{1}{4\pi\varepsilon_0}\frac{qd}{a^3} = \frac{p}{4\pi\varepsilon_0 a^3}$$

$$\alpha = 4\pi\varepsilon_0 a^3 = 3\varepsilon_0 v$$ (v is the volume of the atom)

In case of a molecule it is more complex, their polarization may not be isotropic:



$\vec{E}$ $\alpha = 2\times10^{-40} c^2 \cdot {}^{m}/_{N}$

$\vec{E}$ $\alpha = 4.5\times10^{-40} c^2 \cdot {}^{m}/_{N}$

Generally:

$$\vec{P} = \alpha_\perp \vec{E}_\perp + \alpha_{||} \vec{E}_{||}$$

the most general linear relation

$$\vec{P} = \vec{\vec{\alpha}} \cdot \vec{E}$$

($\vec{\vec{\alpha}}$ is polarizability tensor)

or in (x,y,z)

$$P_x = \alpha_{xx} E_x + \alpha_{xy} E_y + \alpha_{xz} E_z$$

$$P_y = \alpha_{yx} E_x + \alpha_{yy} E_y + \alpha_{yz} E_z$$

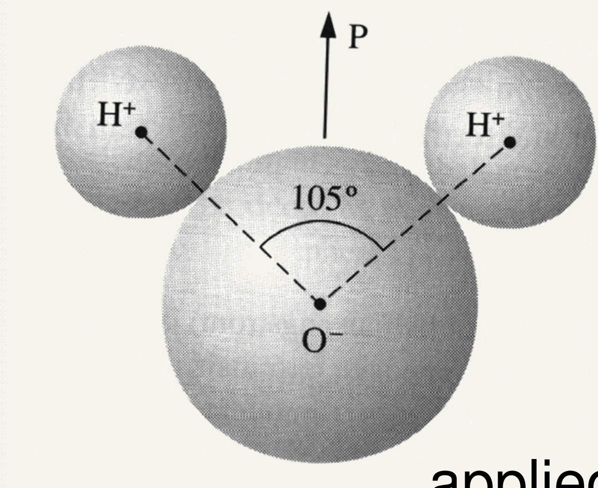
$$P_z = \alpha_{zx} E_x + \alpha_{zy} E_y + \alpha_{zz} E_z$$

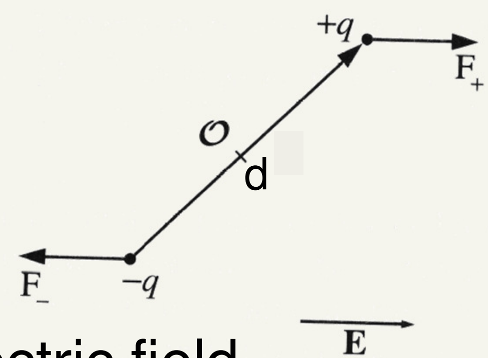
α_{ij} depend on the orientation of the axis you chose.

It's possible to select axes such that $\alpha_{ij} = 0$ for $i \neq j$

4.1.3 Alignment of Polar Molecules

Polar molecules have built-in, permanent dipole moments





applied electric field
(uniform)

Torque $\vec{N} = \left(\vec{r}_+ \times \vec{F}_+\right) + \left(\vec{r}_- \times \vec{F}_-\right)$

$$= \left[\left(\frac{\vec{d}}{2}\right) \times \left(q\vec{E}\right)\right] + \left[\left(-\frac{\vec{d}}{2}\right) \times \left(-q\vec{E}\right)\right] = q\vec{d} \times \vec{E}$$

$$\boxed{\vec{N} = \vec{P} \times \vec{E}} \qquad \left(q\vec{d} = \vec{P}\right)$$

4.1.3

If the applied electric field is nonuniform,
the total force is not zero.

$$\vec{F} = \vec{F}_+ + \vec{F}_- = q\left(\vec{E}_+ - \vec{E}_-\right) = q\left(\Delta\vec{E}\right)$$

for short dipole,

$$dE_x = \left(\nabla E_x\right)\cdot\vec{d} = d_x \frac{\partial E_x}{\partial x}$$

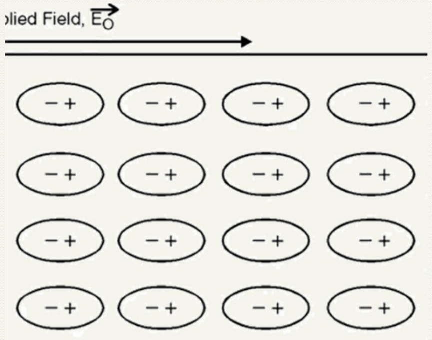
$$dE_y = \left(\nabla E_y\right)\cdot\vec{d} = d_y \frac{\partial Ey}{\partial y}$$

$$dE_z = \left(\nabla E_z\right)\cdot\vec{d} = d_z \frac{\partial E_z}{\partial z}$$

$$d\vec{E} = \left(\vec{d}\cdot\nabla\right)\vec{E}$$

$$\vec{F} = \left(\vec{P}\cdot\nabla\right)\vec{E}$$

4.1.4 Polarization and Susceptibility



The polarization of a polarized dielectric

$\vec{P} \equiv$ dipole moment per unit volume

$\vec{P} = \varepsilon_0 \overleftrightarrow{X}_e \cdot \vec{E}$ $\quad\quad$ $\overleftrightarrow{X}_e$ is electric susceptibility tensor

$$P_x = \varepsilon_0 (X_{e_{xx}} E_x + X_{e_{xy}} E_y + X_{e_{xz}} E_z)$$
$$P_y = \varepsilon_0 (X_{e_{yx}} E_x + X_{e_{yy}} E_y + X_{e_{yz}} E_z)$$
$$P_z = \varepsilon_0 (X_{e_{zx}} E_x + X_{e_{zy}} E_y + X_{e_{zz}} E_z)$$

for linear dielectric

$\vec{P} = \varepsilon_0 \chi_e \vec{E}$ $\quad\quad$ X_e is electric susceptibility

4.2 The Field of a Polarized Object

4.2.1 Bound charges

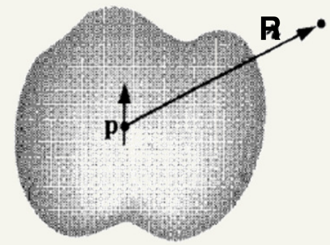
4.2.2 Physical Interpretation of Bound Charge

4.2.3 The Field Inside a Dielectric

4.2 The Field of a Polarized Object

Bound charges:

Divide the material into infinitesimal dipoles and sum up



$$V = \frac{1}{4\pi\varepsilon_0}\int_V \vec{P}\cdot\frac{\hat{R}}{R^2}d\tau = \frac{1}{4\pi\varepsilon_0}\int_V \vec{P}\cdot\nabla(\frac{1}{R})d\tau$$

$$= \frac{1}{4\pi\varepsilon_0}[\int_V \nabla\cdot\left(\frac{1}{R}\vec{P}\right)d\tau - \int_V \frac{1}{R}\left(\nabla\cdot\vec{P}\right)d\tau]$$

$$= \frac{1}{4\pi\varepsilon_0}\int_{surf}\frac{1}{R}\vec{P}\cdot d\vec{a} - \frac{1}{4\pi\varepsilon_0}\int_V \frac{1}{R}\left(\nabla\cdot\vec{P}\right)d\tau$$

bound charges: surface charge $\sigma_b = \vec{P}\cdot\hat{n}$

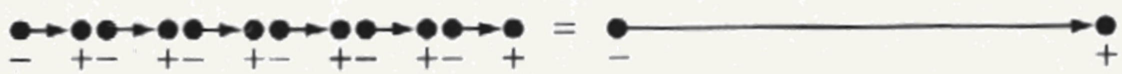
volume charge $\rho_b = -\nabla\cdot P$

$$V = \frac{1}{4\pi\varepsilon_0}\int_{surf}\frac{1}{R}\sigma_b da + \frac{1}{4\pi\varepsilon_0}\int_V \frac{1}{R}\rho_b d\tau$$

4.2.2 Physical Interpretation of Bound Charge

ρ_b and σ_b represent perfectly genuine accumulations of charge.

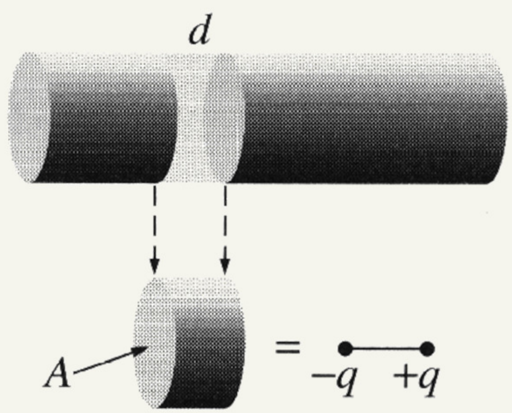
for uniformly distributed dipoles



σ_b

$\rho_b = 0$

for a uniform polarization and perpendicular cut



dipole moment $= P(Ad) = qd$

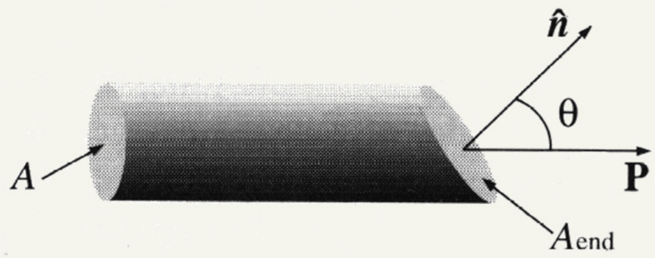
$q = PA$

$\sigma_b = \dfrac{q}{A} = P$

S

4.2.2 (2)

for a uniform polarization and oblique cut



$$q = PA$$

$$\sigma_b = \frac{q}{A_{end}} = P \cdot \cos\theta = \vec{P} \cdot \hat{n}$$

$$\sigma_b = \vec{P} \cdot \hat{n} \qquad \rho_b = ?$$

If the polarization is nonuniform ,

$$\delta q = -\frac{\delta P}{\delta x} \qquad \rho_b d\tau = -\frac{\delta}{\delta x}(P d\tau)$$

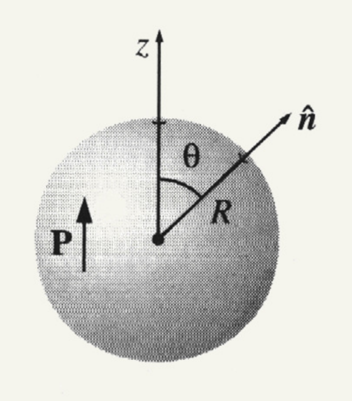
$$- +$$

$$- + - + - +$$

$$\underbrace{- + - +}_{-}\;\underbrace{- +}_{-}\;\underbrace{- +}_{+}\;\underbrace{- +}_{+}$$

In 3D this is $\rho_b = -\nabla \cdot \vec{P}$

Example: Find the field of a uniformly polarized sphere of radius R

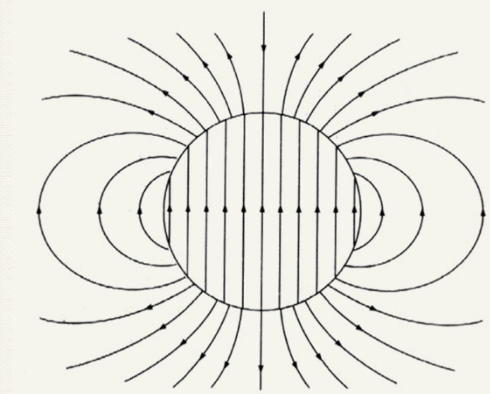


$$\rho_b = \nabla.\vec{P} = 0 \qquad \sigma_b = \vec{P}\cdot\hat{n} = P\cos\theta$$

Ex.9 of chapter 3

$$V(r,\theta) = \begin{cases} \dfrac{P}{3\varepsilon_0} r\cos\theta & \text{for } r \le R \\ \dfrac{P}{3\varepsilon_0}\dfrac{R^3}{r^2}\cos\theta & \text{for } r \ge R \end{cases}$$

$$\because r\cos\theta = z$$



$$\text{for} \quad r \le R \quad V = \frac{P}{3\varepsilon_0} z$$

$$\vec{E} = -\nabla V = -\frac{P}{3\varepsilon_0}\hat{z} = -\frac{1}{3\varepsilon_0}\vec{P}$$

$$\text{for} \quad r \ge R \qquad V = \frac{1}{4\pi\varepsilon_0}\frac{4\pi}{3}R^3\frac{1}{r^2}\vec{p}\hat{r}$$

Similar to a potential of a dipole moment $\vec{p} = \dfrac{4\pi}{3}R^3\vec{P}$

4.3 The Electric Displacement

Gauss' Law in The Presence of Dielectrics:

Total charge density $\rho = \rho_b + \rho_f$

$\uparrow$ bound $\quad \uparrow$ free

Gauss' Law
$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \varepsilon_0 \nabla \cdot \vec{E} = \rho = \rho_b + \rho_f = -\nabla \cdot \vec{P} + \rho_f$$

$$\Rightarrow \nabla \cdot (\varepsilon_0 \vec{E} + \vec{P}) = \rho_f$$

So in a dielectric medium Gauss' law can be written as

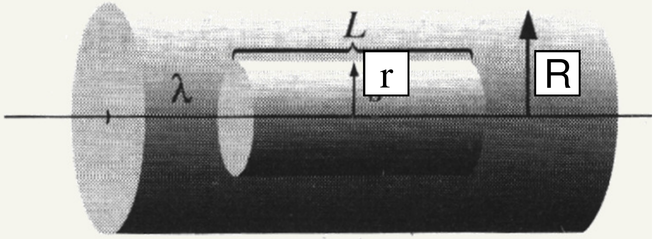
$$\nabla \cdot \vec{D} = \rho_f$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

electric displacement

$$\oint_{surface} \vec{D} \cdot d\vec{a} = Q_{f_{enc.}}$$

Example: A long wire with linear charge λ is covered with an insulation of radius *a.* Find the electrical displacement.



$$\oint_{surface} \vec{D} \cdot d\vec{a} = Q_{f_{enc.}}$$

$$D \cdot (2\pi r L) = \lambda L$$

$$\vec{D} = \frac{\lambda}{2\pi r} \hat{r}$$

inside $\vec{E} = ?$ $\because$ need to know $\vec{P}$

outside $\vec{E} = \frac{1}{\varepsilon_0} \vec{D} = \frac{\lambda}{2\pi \varepsilon_0 r} \hat{r}$ $for \quad r > R$

4.3.2 A Deceptive Parallel

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \Rightarrow \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0}\int \rho \frac{\hat{R}}{R^2} d\tau$$

$$\Downarrow$$

$$\nabla \times \vec{E} = 0$$

since $\nabla \times \vec{A} = 0$ this solution is unique

(Helmhortz theorem: $\vec{V}(r) = \nabla(\nabla \cdot \vec{V}(r)) - \nabla \times (\nabla \times \vec{V}(r))$

But $\nabla \times \vec{D} = \varepsilon_0 (\nabla \times \vec{E}) + \nabla \times P \qquad \nabla \times \vec{P}$ is not always zero

$$\nabla \cdot \vec{D} = \rho_f \quad \text{but} \quad \vec{D} \neq \frac{1}{4\pi}\int \rho_f \frac{\hat{R}}{R^2} d\tau$$

And in general there is no potential for D (cannot be written as a gradient of a scalar)

Permittivity, Dielectric Constant:

for linear dielectrics $\vec{P} = \varepsilon_0 \chi_e \vec{E}$

χ_e Electric susceptibility

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi_e \vec{E} = \varepsilon_0 (1 + \chi_e) \vec{E}$$

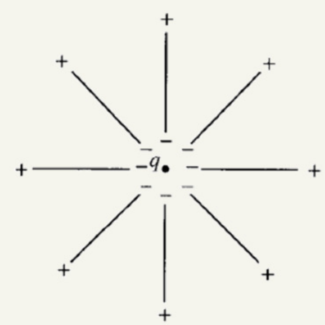
$$\vec{D} = \varepsilon \vec{E} \qquad \varepsilon = \varepsilon_0 (1 + \chi_e)$$

permittivity (ε) permittivity of free space (ε_0)

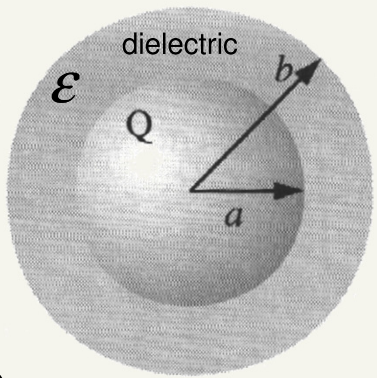
dielectric constant or relative permittivity $\varepsilon_r = 1 + \chi_e = \dfrac{\varepsilon}{\varepsilon_0}$

$$\rho_b = -\nabla \cdot \vec{P} = -\nabla \cdot (\frac{\varepsilon_0 \chi_e}{\varepsilon} \vec{D}) = -(\frac{\chi_e}{1 + \chi_e}) \nabla \cdot \vec{D} = -(\frac{\chi_e}{1 + \chi_e}) \rho_f$$

$$\rho = \rho_b + \rho_f = \left(1 - \frac{\chi_e}{1 + \chi_e}\right) \rho_f = \frac{\rho_f}{1 + \chi_e} = \frac{\rho_f}{\varepsilon_r}$$



Example: A metal sphere of radius a has a charge Q. It is surrounded by a liner dielectric material of permittivity ε . Find the potential at the center, and bound charges of the medium (ρ_b , σ_b).



$$V_{center} = \quad ?$$

$$\vec{E} = \vec{P} = \vec{D} = 0 \qquad \textit{for } r < a$$

$$\vec{D} = \frac{Q}{4\pi r^2}\hat{r} \qquad \textit{for } r > a$$

$$\vec{E} = \begin{cases} \dfrac{Q}{4\pi\varepsilon r^2}\hat{r} & \textit{for } b > r > a \\ \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r} & \textit{for } r > b \end{cases}$$

$$V_{center} = V_a = -\int_{\infty}^{a} \vec{E}\cdot d\vec{\ell} = -\int_{\infty}^{b} \frac{Q}{4\pi\varepsilon_0 r^2} dr - \int_{b}^{a} \frac{Q}{4\pi\varepsilon\ r^2} dr$$

$$= \frac{Q}{4\pi}\left(\frac{1}{\varepsilon_0 b} + \frac{1}{\varepsilon a} - \frac{1}{\varepsilon b}\right)$$

$$\vec{P} = \varepsilon_0 \chi_e \vec{E} = \frac{\varepsilon_0 \chi_e Q}{4\pi\varepsilon r^2}\hat{r}$$

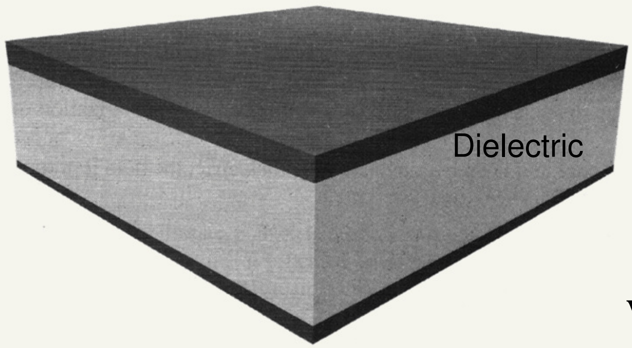
for b > r > a

$$\rho_b = -\nabla \cdot \vec{P} = 0$$

$$(\because \nabla \cdot \vec{P} = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 P_r + \cdots\cdots = 0)$$

$$\sigma_b = \vec{P} \cdot \hat{n} = \begin{cases} \dfrac{\varepsilon_0 x_e Q}{4\pi\varepsilon b^2} & \text{at } r = b \\ -\dfrac{\varepsilon_0 x_e Q}{4\pi\varepsilon a^2} & \text{at } r = a \end{cases}$$

Example: A parallel plate capacitor is filled with an insulating material of dielectric constant ε. What is the effect of the dielectric on its capacitance?



without dielectric $V_{vac} = E.d$

the dielectric reduces E to $\frac{E}{\varepsilon_r}$ so $V_{d.e.} = \frac{E}{\varepsilon} d$

since $C = \frac{Q}{V}$ $\quad C_{d.e.} = \varepsilon_r C_{vac}$

Problem 4.18 The space between the plates of a parallel-plate capacitor (Fig. 4.24) is filled with two slabs of linear dielectric material. Each slab has thickness a, so the total distance between the plates is $2a$. Slab 1 has a dielectric constant of 2, and slab 2 has a dielectric constant of 1.5. The free charge density on the top plate is σ and on the bottom plate $-\sigma$.

(a) Find the electric displacement **D** in each slab.

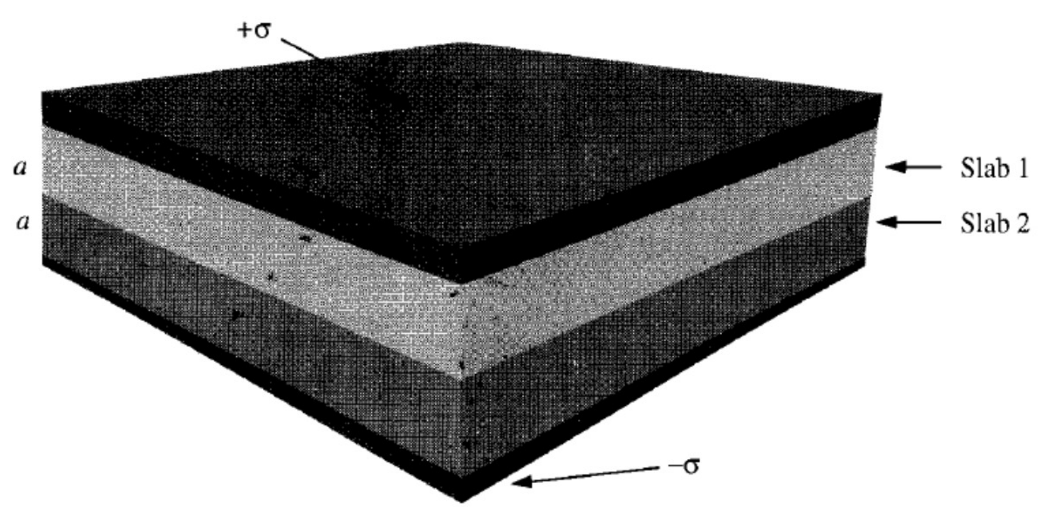
(b) Find the electric field **E** in each slab.

(c) Find the polarization **P** in each slab.

(d) Find the potential difference between the plates.

(e) Find the location and amount of all bound charge.

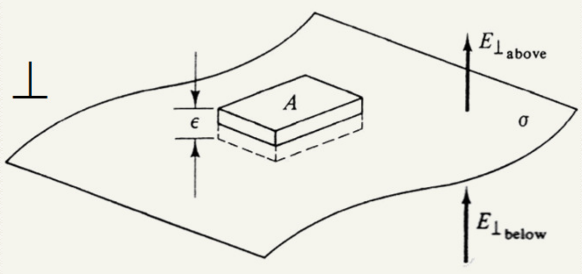
(f) Now that you know all the charge (free and bound), recalculate the field in each slab, and confirm your answer to (b).



Boundary Conditions:

Field normal (⊥) to surface:

⊥

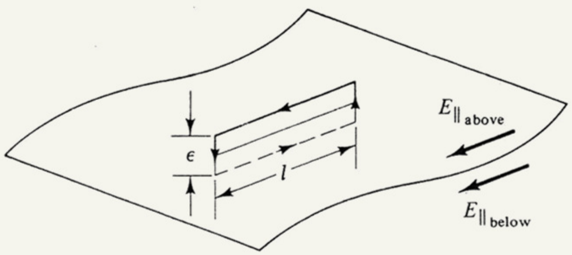


$$\oint_{surface} \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$= E_{\perp above}\cdot A - E_{\perp below}\cdot A$$

$$E_{\perp above} - E_{\perp below} = \frac{\sigma}{\varepsilon_0}$$

Field parallel (||) to surface:



$$\oint \vec{E}\cdot d\vec{l} = 0 \qquad \left(ie.\ \nabla\times\vec{E} = 0\right)$$

$E_{above} = E_{below}$

$$\boxed{E_{||above} - E_{||below}}$$

In case of D above conditions become:

$$D_{\perp above} - D_{\perp below} = \sigma_f \;\Rightarrow\; \varepsilon_{above}\frac{\partial V_{above}}{\partial n} - \varepsilon_{below}\frac{\partial Vbelow}{\partial n} = \sigma_f$$

$$\boxed{D_{||above} - D_{||below} = P_{||above} - P_{||below}}$$

$$\boxed{V_{above} = V_{below}}$$

Example: Dielectric sphere in a uniform electric field, find the electric field inside the sphere.

$$V_{in} = V_{out} \qquad \text{at } r = R$$

$$\varepsilon \frac{\partial V_{in}}{\partial r} = \varepsilon_{out} \frac{\partial V_{out}}{\partial r} \qquad \text{at } r = R \text{ (no free charges at the surface)}$$

$$V_{out} \to -E_0 z = -E_0 \cos\theta \quad \text{for } \; r >> R$$

inside the sphere: $$V(r,\theta) = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta)$$

outside the sphere: $$V(r,\theta) = -E_0 R \cos\theta + \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta)$$

boundary condition (i) $$\sum_{l=0}^{\infty} A_l R^l P_l(\cos\theta) \quad = -E_0 R \cos\theta + \sum_{l=0}^{\infty} \frac{B_l}{R^{l+1}} P_l(\cos\theta)$$

so $A_1 R = -E_0 R + \dfrac{B_1}{R^2}$ and $A_l R^l = \dfrac{B_l}{R^{l+1}}$ for $l \neq 1$

boundary condition (i) $\sum_{l=0}^{\infty} A_l R^l P_l(\cos\theta) \quad = -E_0 R\cos\theta + \sum_{l=0}^{\infty} \frac{B_l}{R^{l+1}} P_l(\cos\theta)$

so $A_1 R = -E_0 R + \frac{B_1}{R^2}$ and $A_l R^l = \frac{B_l}{R^{l+1}}$ for $1 \neq 1$

boundary condition (ii) $\varepsilon_r \sum_{l=0}^{\infty} l A_l R^{l-1} P_l(\cos\theta) \quad = -E_0 \cos\theta - \sum_{l=0}^{\infty} \frac{(l+1)B_l}{R^{l+1}} P_l(\cos\theta)$

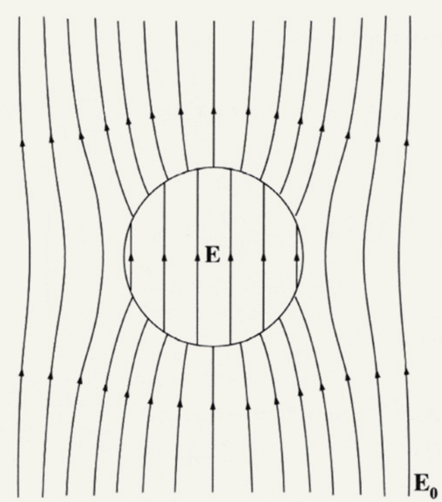
so $\varepsilon_r A_1 = -E_0 - \frac{2B_1}{R^3}$ and $\varepsilon_r l A_l R^{l-1} = -\frac{(l+1)B_l}{R^{l+2}}$ for $1 \neq 1$

$$\varepsilon_r l A_l R^l = -\frac{(l+1)B_l}{R^{l+1}} \Rightarrow \varepsilon_r l = -(l+1) \text{ for } 1 \neq 1$$

this can not be true, so $A_l = B_l = 0$ for $1 \neq 1$;

$$A_1 = -\frac{3E_0}{\varepsilon_r + 2} r\cos\theta, \quad A_1 = \frac{\varepsilon_r - 1}{\varepsilon_r + 2} R^3 E_0$$

$$V_{in}(r,\theta) = -\frac{3E_0}{\varepsilon_r + 2} r\cos\theta = -\frac{3E_0}{\varepsilon_r + 2} z \quad \Rightarrow \vec{\mathrm{E}}(r,\theta) = \frac{3\vec{E}_0}{\varepsilon_r + 2}$$



4.4.3 Energy in a Dielectric System

Energy of a charged capacitor $W = \frac{1}{2}CV^2$

If the capacitor is filled with a dielectric $C = \varepsilon_r C_{vac}$

So the stored energy should increases by the same factor:

energy stored in any electrostatic system (vacuum) $W = \frac{\varepsilon_0}{2}\int E^2 d\tau$

with linear dielectric $$W = \varepsilon_r\left(\frac{\varepsilon_0}{2}\int E^2 d\tau\right) = \frac{1}{2}\int \vec{D}\cdot\vec{E}\,d\tau$$

which suggests the energy in the dielectric is $= \frac{1}{2}\int \vec{D}\cdot\vec{E}\,d\tau$

4.4.3 Energy in Dielectric media (formal proof)

Suppose we are gradually building up charge in a dielectric medium

$$\delta W = \int (\delta\rho_f) V d\tau \qquad \nabla\cdot\vec{D} = \rho_f \Rightarrow \Delta\rho_f = \nabla\cdot\Delta\vec{D}$$

$$= \int (\nabla\cdot\delta\vec{D}) V d\tau \qquad \nabla\cdot(\delta\vec{D}V) = (\nabla\cdot\delta\vec{D})V + \delta\vec{D}\cdot(\nabla V)$$

$$= \int \nabla\cdot(\delta\vec{D}V) d\tau - \int \delta\vec{D}\cdot\nabla V d\tau$$

$$= \int_s \delta\vec{D}V\cdot d\vec{a} + \int \delta\vec{D}\cdot\vec{E} d\tau$$

$\searrow$ 0 for $S \to \infty$

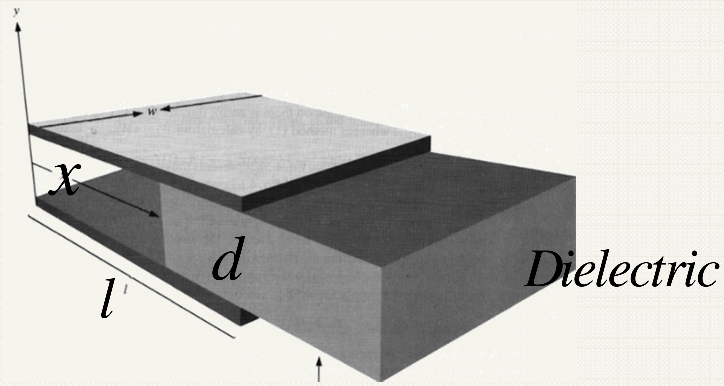
for linear dielectric, $\vec{D} = \varepsilon\vec{E} \Rightarrow \delta\vec{D}\cdot\vec{E} = \delta\varepsilon\vec{E}\cdot\vec{E} = \frac{1}{2}\delta(\varepsilon E^2) = \frac{1}{2}\delta(\vec{D}\cdot\vec{E})$

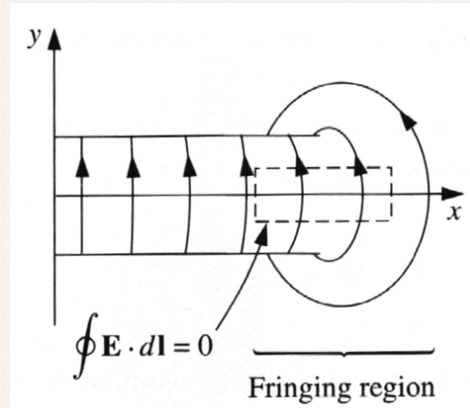
$$= \int \delta(\tfrac{1}{2}\vec{D}\cdot\vec{E}) d\tau$$

$$= \delta[\tfrac{1}{2}\int \vec{D}\cdot\vec{E} d\tau]$$

$$\therefore \quad W = \frac{1}{2}\int \vec{D}\cdot\vec{E} d\tau$$

4.4.4 Forces on dielectrics





with Q constant $dW = F_{ex} dx$

$$W = \frac{1}{2} C V^2 = \frac{1}{2} \frac{Q^2}{C} \qquad \vec{F} = -F_{ex} = -\frac{dW}{ds} = \frac{1}{2} \frac{Q^2}{C^2} \frac{dC}{dx} = \frac{1}{2} V^2 \frac{dC}{dx}$$

$$C = \frac{Q}{V} = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 w}{d} (\varepsilon_r \cdot l - \chi_e x) \Rightarrow \frac{dC}{dx} = -\frac{\varepsilon_0 X_e w}{d}$$

$$\vec{F} = -\frac{\varepsilon_0 \chi_e w}{2d} V^2$$

note we used $W = \frac{1}{2} \frac{Q^2}{C}$ not $W = \frac{1}{2} C V^2$

If V is kept constant:

If $W = \frac{1}{2}CV^2$ used $\quad \vec{F} = -F_{ex} = -\frac{dW}{dx} = -\frac{1}{2}V^2\frac{dC}{dx}$ wrong sign

The reason is to keep V constant the battery must do work.
So that has to be taken in to account

$$dW = F_{ex}dx + VdQ$$

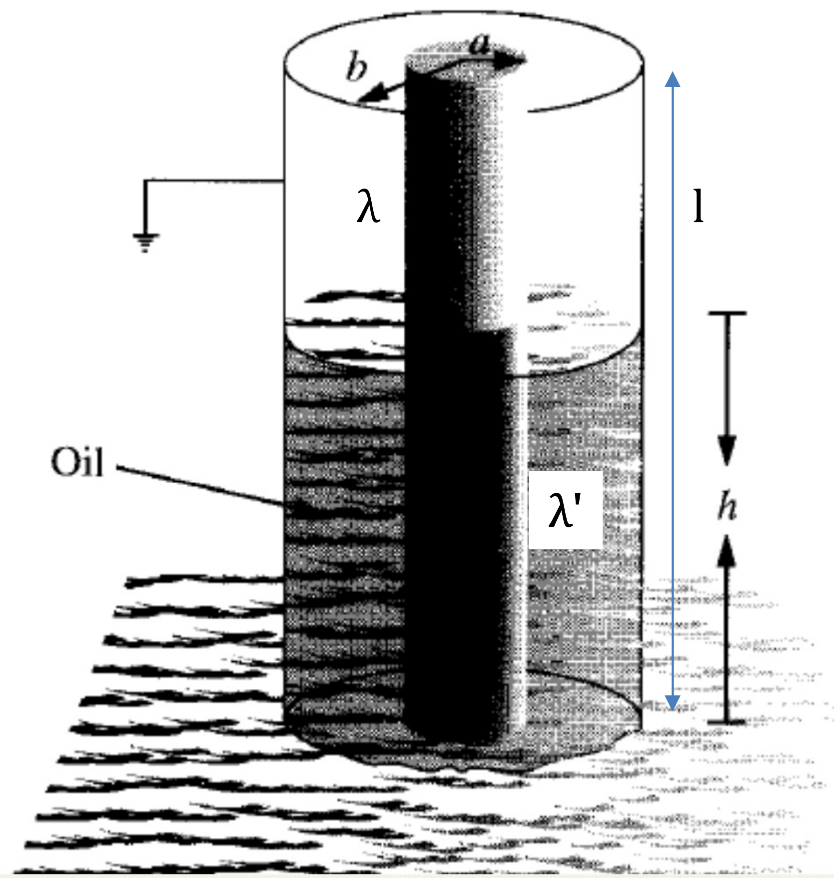
$$\vec{F} = -F_{ex} = -\frac{dW}{dx} + V\frac{dQ}{dx}$$

$W = \frac{1}{2}CV^2 \Rightarrow$

$$= -\frac{1}{2}V^2\frac{dC}{dx} + V^2\frac{dC}{dx}$$

$$= \frac{1}{2}V^2\frac{dC}{dx}$$ now with correct sign/direction

Problem 4.28 Two long coaxial cylindrical metal tubes (inner radius a, outer radius b) stand vertically in a tank of dielectric oil (susceptibility χ_e, mass density ρ). The inner one is maintained at potential V, and the outer one is grounded (Fig. 4.32). To what height (h) does the oil rise in the space between the tubes?



Hint: find the capacitance of tube first