Phys 503/729 – General Relativity and Gravitation – Fall 2015
Homework Assignment #6, due Monday, November 9

In your solutions to the following problems, please include full sentences explaining all of the steps in your reasoning. A solution that includes only equations or very few words will not be considered complete.

Problem #1 (Phys 503 and 729)

Carroll, Chapter 5, Exercise 3.

Problem #2 (Phys 503)

Using the components of the Riemann tensor given in Carroll's book by Eq. 5.12, with $\mathrm{e}^{2\alpha} = 1 - 2GM/r$ and $\beta = -\alpha$, show that

$$R^{\mu\nu\sigma\rho}\, R_{\mu\nu\sigma\rho} = \frac{48\, G^2 M^2}{r^6} ,$$

as stated in Eq. 5.50.

Problem #2 (Phys 729)

Carroll, Chapter 5, Exercise 4.

Problem #3 (Phys 503)

(Hartle, Problem 9.1) An advanced civilization living outside a spherical neutron star of mass M constructs a massless shell concentric with the star such that the area of the innner surface is $144\pi\, M^2$ and the area of the outer surface is $400\pi\, M^2$. What is the physical thickness of the shell?

Problem #3 (Phys 729)

(Wald, Problem 6.1) Let M be a 3-dimensional manifold possessing a spherically symmetric Riemannian metric with $\nabla_a r \neq 0$, where r is related to the area A of a sphere of constant r by $A = 4\pi r^2$.
(a) Show that a new "isotropic" radial coordinate $\tilde{r}$ can be introduced so that the metric takes the form

$$\mathrm{d}s^2 = H(\tilde{r})[\mathrm{d}\tilde{r}^2 + \tilde{r}^2\, \mathrm{d}\Omega^2] .$$

(Thus, every spherically symmetric 3-dimensional space is conformally flat.)
(b) Show that in isotropic coordinates the Schwarzschild metric is

$$\mathrm{d}s^2 = -\frac{(1 - GM/2\tilde{r})^2}{(1 + GM/2\tilde{r})^2}\, \mathrm{d}t^2 + \left(1 + \frac{GM}{2\tilde{r}}\right)^4 [\mathrm{d}\tilde{r}^2 + \tilde{r}^2 \mathrm{d}\Omega^2] .$$