1-2 Measurement and Units

"The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding, can lead to them."

—ALBERT EINSTEIN

Before you can figure out the laws or rules things follow, you must first observe them. To develop a "sympathetic understanding," you need to become familiar with how things behave. Physics necessarily begins with what we detect by means of our senses. But then, to develop a shared understanding, we must be able to agree in detail on what we have observed and we must be able to communicate what we see without risk of being misunderstood. It is too vague, for example, to say that a ball player is large. Which athlete in Figure 1-5 is larger? By what standard? It would be clearer to give each athlete's height, weight, or shoulder width. We could agree on how the measurements compare, no matter which athlete we call larger.

Figure 1-5 **Comparing the "largeness" of athletes.** Who is "larger," the tall, thin basketball player or the shorter but much broader football player?

In physics, therefore, observations are generally **quantitative,** that is, they are expressed in terms like *height* and *weight* that can have numerical values. Something that can have a numerical value is called a **quantity.** Speed, area, and the price per pound of potatoes are all examples of quantities. For the remainder of this chapter, we will focus on some aspects of how we treat quantities. As we do so, bear in mind Einstein's emphasis on intuition and a sympathetic understanding of how things behave. Physics is not just mathematics, or even primarily mathematics. Your use of mathematics has to be guided by thinking about how things behave and what rules or physical principles govern their behavior.

Measurements are quantitative observations made in comparison with a **standard,** which we call a **unit of measurement.** For many years, the distance between two fine lines engraved on a bar of platinum-iridium alloy kept at the International Bureau of Weight and Measures outside Paris was the internationally recognized standard meter. The standard **meter** is now defined as the length of the path traveled by light in a vacuum during a time interval of 1/299

792 458 of a second. When we say a soccer field is 100 meters long, we mean it is 100 times as long as the carefully marked-off unit called a meter.

Likewise, we can establish a unit of mass by choosing a particular block of metal: a cylinder of platinum-iridium alloy serves this purpose at the International Bureau of Weights and Measures. Another mass is equal to this mass if it just balances it on an equal arm balance in a uniform gravitational environment. If we call our standard unit of mass a **kilogram,** a rat will have a mass of 3 kg if it just balances three of these units on a balance scale (Figure 1-6). For now, what we will mean by the mass of an object is the number of standard units that it can counterbalance on an equal arm balance. Physicists call this an **operational definition,** because we are defining mass by what we do (the "operation" we perform) to measure it.

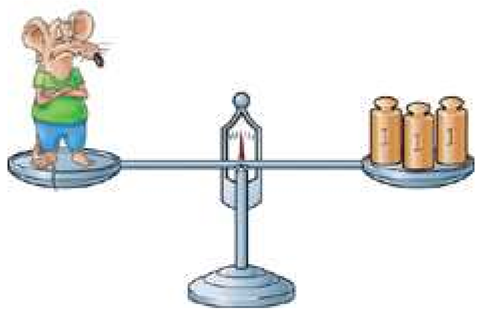

Figure 1-6 **Balancing a laboratory rat.** The three standard 1-kg masses just balance a (very large) 3-kg laboratory rat.

Crudely speaking, mass measured in this way gives us a feel for "how much stuff" we have. But mass is different from weight. Placed on the moon, the contents of each pan in Figure 1-6 will weigh less, but the rat still counterbalances the three standard kilograms and thus still has a mass of 3 kg.

To measure **time duration,** we must choose the duration of some particular happening as our standard or unit. But we cannot pick something that happens just once, because we could never go back and check it. How would we know if our clock has sped up if the standard is gone? We therefore have to pick a happening that keeps repeating itself, such as the back-and-forth swing of a pendulum. Such occurrences are said to be **cyclical** or **periodic.** Two well-known examples of periodic occurrences or cycles used as standard units of time duration are the duration of a complete rotation of Earth on its axis, which we call a day, and the duration of one complete orbit of Earth about the sun, which we call a year. These and other units are now taken as multiples of the **second,** which is itself defined as a multiple of a cycle characteristic of a particular type of radiation emitted by cesium atoms.

The units most commonly used in physics are the units of the Système Internationale (**SI units**), a current version of the metric system generally agreed on by the international scientific community and in extensive everyday use in nearly every country in the world except the United States. The basic SI units for fundamental quantities, including those we have considered so far, are listed in Table 1-1.

Table 1-1 Basic SI units for Fundamental Quantities

Quantity	SI Unit
Distance	meter (m)
Mass	kilogram (kg)

Quantity	SI Unit
Time duration	second (s)
Electric current	ampere (A)
Temperature	kelvin (K)
Amount of substance	mole (mol)
Luminous intensity	candela (cd)

SI units are sometimes called *mks* (meter-kilogram-second) units. Other larger or smaller units of these quantities are expressed as multiples of basic units by a system of prefixes. These prefixes, which represent multiplication by different powers, are summarized in Table 1-2. For instance, 1 nanosecond is 1×10^{-9} seconds, and 5 kilograms is 5×10^{3} grams. In the latter case, it is the kilogram that we take as basic, not the gram.

Table 1-2 Prefixes for SI (or Metric) Units

The Prefix …	Is Abbreviated …	And Means …	The Prefix …	Is Abbreviated …	And Means …
yetta-	Y	10^{24}	centi-	c	10^{-2}
zetta-	Z	10^{21}	milli-	m	10^{-3}
exa-	E	10^{18}	micro-	μ (mu)	10^{-6}
peta-	P	10^{15}	nano-	n	10^{-9}
tera-	T	10^{12}	pico- or	p	10^{-12}
giga-	G	10^{9}	micromicro-	or $\mu\,\mu$	
mega-	M	10^{6}	femto-	f	10^{-15}
kilo-	k	10^{3}	atto-	a	10^{-18}
hecto-	h	10^{2}	zepto-	z	10^{-21}
deka-	da	10	yocto-	y	10^{-24}
deci-	d	10^{-1}			

By basic units, we mean that units of all other quantities can be defined in terms of these. In contrast to basic units, those are called *derived units.* For instance, the unit of *two-dimensional space* or **area** is a square 1 m by 1 m, called a **square meter.** A rectangle measuring 3 m by 2 m (Figure 1-7*a*) thus has an area equal to length × width = 3 m × 2 m = 6 m^2 because there are three rows of two square-meter squares in this rectangle. When we multiply units as well as numbers, we get m^2 as units of area. To make this meaningful, we choose to identify 1 "m^2" as a square meter.

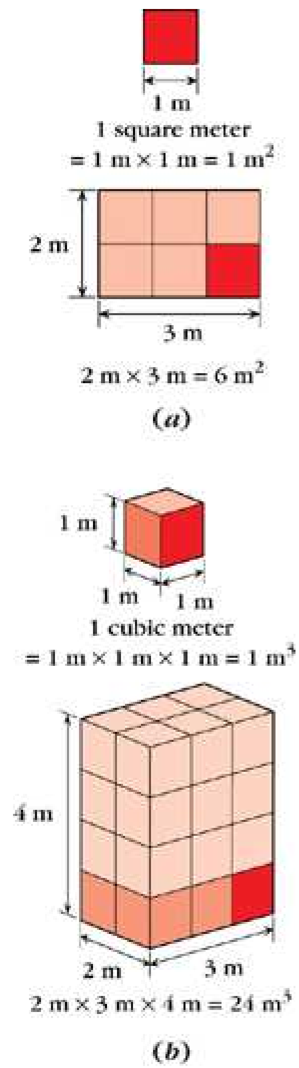

Figure 1-7 Units of area and volume.

We can similarly derive units of three-dimensional space (**volume**). If we have a block measuring 3 m by 2 m by 4 m (Figure 1-7*b*), we can picture it as made up of cubes 1 m on a side, which we call **cubic meters.** As the figure shows, each layer has three rows of two cubes (six cubes in all), and there are four layers, so in all there are $3 \times 2 \times 4 = 24$ cubic meters. In effect, we have multiplied length by width by height to get volume $V = lwh$). Multiplying units as well as numbers gives $V = 3\text{ m} \times 2\text{ m} \times 4\text{ m} = 24\text{ m}^3$. This is meaningful only if we identify m^3 as a cubic meter.

To get a volume in m^3, length, width, and height must all be in meters. This is always the case: To get a derived quantity in standard units, the quantities you use to calculate it must be in standard units. Some derived units have names that obscure their derivation. It will turn out, for example, that the SI unit of energy is $1 \dfrac{\text{kg} \times \text{m}^2}{s^2}$ which is called a joule. In energy calculations, your energy will not come out in joules unless you are working with mass in kilograms, length in meters, and time duration in seconds.

The basic quantities involved in the definition of a derived quantity are called its *dimensions.* If we represent the basic quantities mass, length, and time duration by the bracketed symbols [M], [L], and [T], then the dimensions of energy are $\left[\frac{\text{ML}}{\text{T}^2}\right]$ or [MLT^{-2}]. Appendix F provides a fuller treatment of dimensions and of a method called **dimensional analysis** for checking dimensions to see whether there is an error in a mathematical relationship among physical quantities.

✦ **Converting Units** When you do a calculation, the available values of quantities are not always in the units you want. In that case, you have to convert units. This is often true outside of physics as well. A change machine, for example, is a device that converts from dollars to quarters. You end up with the *same value,* but expressed in different units.

To convert units, you first need a conversion relationship, such as "one dollar equals four quarters" or 1 min = 60 s. Dividing both sides of the equation by the same thing, you can arrive at either

$$\frac{1\ \text{min}}{60\ \text{s}} = \frac{60\ \text{s}}{60\ \text{s}} = 1 \quad \text{or} \quad 1 = \frac{1\ \text{min}}{1\ \text{min}} = \frac{60\ \text{s}}{1\ \text{min}}$$

Thus you can write one (1) as either $\frac{1\ \text{min}}{60\ \text{s}}$ or $\frac{60\ \text{s}}{1\ \text{min}}$. Multiplying by one never changes the value of something. When we convert, we want to change the units in which a value is expressed without changing the value. We can do that by multiplying by one (1) written in suitable form:

$$\text{To convert 5 min to seconds:} \qquad 5\ \text{min} = 5\ \cancel{\text{min}} \times \frac{60\ \text{s}}{1\ \cancel{\text{min}}} = 300\ \text{s}$$

$$\text{To convert 300 s to min:} \qquad 300\ \text{s} = 300\ \cancel{\text{s}} \times \frac{1\ \text{min}}{60\ \cancel{\text{s}}} = 5\ \text{min}$$

Like one dollar and four quarters, 300 s is the same value as 5 min, but expressed in different units. In each case, we pick the form of one so that when we multiply, the units we don't want "cancel out," and we are left with the units we do want.

Procedure 1-1

Converting a Quantity to Different Units

➥ **Caution:** Notice that the numerical value is different in different units. It is therefore meaningless to give a numerical value for a quantity without giving its units as well.

1. Rewrite your conversion relationship (x first units = y second units) as either

$$\frac{x\ \text{first units}}{y\ \text{second units}} = 1 \quad \text{or} \quad \frac{y\ \text{second units}}{x\ \text{first units}} = 1$$

2. Multiply the quantity by 1 in whichever of these two forms cancels out the units you don't want and leaves the units you do want in the right place (numerator or denominator).

3. Check to make sure your result makes sense: You should always get more of the smaller unit, fewer of the larger unit.

Example 1-1 *Converting Speed*

For a guided interactive solution, go to Web Example 1-1 at www.wiley.com/college/touger

A bus travels 110 km/h (kilometers per hour) on open highway. What is this speed in standard SI units?

Brief Solution

1. Identify the units you want for your answer. In SI, distances are in meters (m) and time durations are in seconds (s).

2. 110 km/h is really a fraction $110\,\frac{\text{km}}{\text{h}}$ or $\frac{110\text{ km}}{1\text{ h}}$, and means 110 kilometers are traveled *in each* hour or *per* hour.

3. Write the conversion relations between the units you start out with and those you want. In this case, it may be easier to convert time in two steps, first from hours to minutes, then from minutes to seconds.

$$1\text{ km} = 1000\text{ m} \quad 1\text{ h} = 60\text{ min} \quad 1\text{ min} = 60\text{ s}$$

4. As fractions, these relations become

$$\frac{1\text{ km}}{1000\text{ m}} = \frac{1000\text{ m}}{1\text{ km}} = 1 \quad \frac{1\text{ h}}{60\text{ min}} = \frac{60\text{ min}}{1\text{ h}} = 1 \quad \frac{1\text{ min}}{60\text{ s}} = \frac{60\text{ s}}{1\text{ min}} = 1$$

5. Multiply by 1 as many times as necessary to get the units you want:

$$110\frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \underbrace{\frac{1000\text{ m}}{1\ \cancel{\text{km}}}}_{\text{"cancels out" km in num., puts m in num.}} \times \underbrace{\frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}}_{\text{"cancels out" h in denom., puts min in denom.}} \times \underbrace{\frac{1\ \cancel{\text{min}}}{60\text{ s}}}_{\text{"cancels out" min in denom., puts s in denom.}} = \frac{110\,000\text{ m}}{3600\text{ s}} = \mathbf{30.6\ m/s}$$

Alternative method. You can also do unit conversion by substitution. For instance, in Example 1-1 you can substitute 1000 m for 1 km, 60 min for 1 h, and 60 s for 1 min. Thus,

$$110\,\frac{\text{km}}{\text{h}} = \frac{110(1000\text{ m})}{60\text{ min}} = \frac{110(1000\text{ m})}{60(60\text{ s})} = 30.6\text{ m / s or } 30.6\text{ m / s}$$

- **Related homework: Problems 1-5, 1-10, 1-11, 1-12, and 1-20.**

Example 1-2 *Buying a Carpet*

You want to carpet a 12 ft × 15 ft room. You can readily calculate that the floor area is 180 ft^2, but carpeting is sold by the square yard (yd^2). How many square yards do you need? **STOP&Think** Since 1 yd = 3 ft, should you just divide by 3? ◆

Solution

1. We have the conversion relation 1 yd = 3 ft, which we can rewrite as $\frac{1 \text{ yd}}{3 \text{ ft}} = 1$ or $\frac{3 \text{ ft}}{1 \text{ yd}} = 1$.

2. Remember that 1 ft^2 = 1 ft × 1 ft. Thus, 180 ft^2 = 180 ft × ft, and we have to end up with yd^2 = yd × yd. We therefore have to multiply twice by $\frac{1 \text{ yd}}{3 \text{ ft}}$:

$$180 \text{ ft}^2 = 180 \text{ ft} \times \text{ft} = 180 \cancel{\text{ft}} \times \cancel{\text{ft}} \times \frac{1 \text{ yd}}{3 \cancel{\text{ft}}} \times \frac{1 \text{ yd}}{3 \cancel{\text{ft}}}$$
$$= 20 \text{ yd} \times \text{yd} = \mathbf{20\ yd^2}$$

◆ **Related homework: Problems 1-6 and 1-9.**

✦ **Significant Figures** No measurement is completely precise. You cannot read distances much smaller than 0.001 m (1 mm) on a meter stick, nor can you read more than a certain number of places on any instrument that has a numerical readout, be it an electronic balance calibrated in units of mass or a multimeter that measures electric current and voltage. The number of places that you can legitimately read with your measuring instrument is called the number of significant figures. A numerical value should always be written to show the number of significant figures. Suppose you measure "exactly" two meters on a tape measure that has 0.001 m accuracy. The measured value is not really exact, but it is closer to 2.000 m than to 2.001 m or to 1.999 m. Therefore, you must write 2.000 m, not 2 m, to represent your measurement. If you converted to kilometers (1 m = 10^{-3} km), you would have to write 2.000×10^{-3} m, not 2×10^{-3} m.

STOP&Think Does a mass of 5000 kg represent one significant figure? Two? Three? Four? More? ◆

Ordinary writing of numbers is sometimes ambiguous, but in scientific notation we can distinguish readily the number of significant figures in 5×10^3 kg (one), 5.000×10^3 kg (four), or 5.000000×10^3 kg (seven, that is, more).

When you use your measured values to calculate a result, *you cannot claim greater accuracy (more significant figures) for your result than for the measurements from which it came.* Suppose A = 2.000 m and B = 3.000 m are the measured lengths of the two legs of a right triangle. You wish to calculate the length of the hypotenuse using the Pythagorean theorem: $A^2 + B^2 = C^2$. Using your calculator, you obtain the value C = 3.605551725 m. The last six places of this calculator readout are meaningless because your measurements could give you only four significant figures. Because your calculator readout is closer to 3.606 than to 3.605, you must write that C = 3.606 m.

If the measurements you had were A = 2.000 m and B = 3.0 m because B was measured by a less precise instrument, you would have to write your result as C = 3.6 m. Your result cannot have more significant figures than *any* of the values you used to find it.

When you *estimate,* you can sometimes be more flexible, because you are basing your calculations on numbers that you either guess at based on experience or round off for convenience. For example, Mrs. Wang knows she can get carpeting for $8.79 a square yard. She eyeballs her children's playroom and says, “This looks to be about 10 feet by 15 feet. That's about 150 square feet. A square yard is around 10 square feet (actually it is 9), so that's about 15 square yards. It's probably a bit more, but if I figure $10 a square yard, that will compensate, so I should budget about $150 ($10 a square yard × 15 square yards) to carpet the room.”

In pursuing its quest of the rules by which nature plays, physics must adhere to the rules and tools of careful logical reasoning. In addressing measurement and units, we have taken a few small steps toward building the rich and varied toolkit that we will need.