2-4 Acceleration and Graphs of Accelerated Motion

We have looked at several situations in which an object's velocity changes over time. We know that an object, in everyday language, can pick up or lose speed. A powerful sports car picks up speed more quickly than an economy car; a braking car loses speed more quickly on dry pavement than on an icy road. It is therefore useful to talk about the *rate* at which it happens. But we want to do so in language consistent with describing motion in terms of *position, displacement,* and *velocity*. We therefore will define *acceleration* as the rate at which velocity changes with time. If an object's velocity is v_1 at instant t_1 and v_2 at instant t_2, then we define *average acceleration* as follows:

Definition: Over the interval $\Delta t = t_2 - t_1$,

$$\textit{average acceleration} \quad \bar{a} \equiv \frac{\Delta v}{\Delta t} \equiv \frac{v_2 - v_1}{t_2 - t_1} \tag{2-7}$$

for one-dimensional motion.

This definition is an abbreviated statement of the following procedure.

Procedure 2-2

Determining Average Acceleration in One Dimension

1. Determine an object's instantaneous velocity v_1 at time t_1.
2. Determine its instantaneous velocity v_2 at later time t_2.
3. Calculate the increments $\Delta v = v_2 - v_1$ and $\Delta t = t_2 - t_1$.
4. Use Equation 2-7 to calculate $\bar{a}$.

Let's see how this definition works in the case of the cart in Figure 2-13*a*. The cart leaves the hand in the downhill direction at $t = 0$, but soon reverses direction because of the suspended weight. Look at its v versus t graph (Figure 2-12*a*). At t_1 its velocity is negative; it is moving to the left. At t_2 its velocity is zero. Thus, over the interval $\Delta t = t_2 - t_1$,

$$\begin{aligned}\bar{a} &\equiv \frac{\Delta v}{\Delta t} \equiv \frac{v_2 - v_1}{t_2 - t_1} = \frac{0 - (-\text{value of } v)}{\text{positive time interval}} = \frac{\text{positive numerator}}{\text{positive denominator}} \\ &= \text{positive value of average acceleration .}\end{aligned}$$

Likewise, during the interval $\Delta t = t_3 - t_2$,

$$\begin{aligned}\bar{a} &\equiv \frac{\Delta v}{\Delta t} \equiv \frac{v_3 - v_2}{t_3 - t_2} = \frac{(+\text{value of } v) - 0}{\text{positive time interval}} = \frac{\text{positive numerator}}{\text{positive denominator}} \\ &= \text{positive value of average acceleration .}\end{aligned}$$

The cart's acceleration is positive during both intervals! That is because its velocity increases when it goes from a negative value (cart going left) to zero as well as when it goes from zero to a positive value (cart going right). If the velocity is increasing, its rate of change must be positive.

By similar arguments, we can conclude that the average acceleration of the cart in Figure 2-13*b* (which goes right immediately after leaving the hand at $t = 0$, but later goes left) is negative throughout. We can now summarize our reasoning:

	Time Interval Δt	Change in Velocity Δv	$\bar{a} = \frac{\Delta v}{\Delta t}$	What Object Is Doing
Cart in Figure 2-13***a***	$t_2 - t_1 = +\text{value}$	$v_2 - v_1 = 0 - \underset{\text{to left}}{(-\text{value})} = +\text{value}$	+	Moving left but slowing down
	$t_3 - t_2 = +\text{value}$	$v_3 - v_2 = \underset{\text{to right}}{(+\text{value})} - 0 = +\text{value}$	+	Moving right and speeding up
Cart in Figure 2-13***b***	$t_2 - t_1 = +\text{value}$	$v_2 - v_1 = 0 - \underset{\text{to right}}{(+\text{value})} = -\text{value}$	−	Moving right but slowing down
	$t_3 - t_2 = +\text{value}$	$v_3 - v_2 = \underset{\text{to left}}{(-\text{value})} - 0 = -\text{value}$	−	Moving left and speeding up

✦ Units of Average Acceleration

Average acceleration is the *rate of change* of velocity. For instance, if a car went from 0 to 60 mi/hr in 5 seconds, it would be gaining an average of 12 mi/hr each second, or per second. Its acceleration in mixed units would be 12 (mi/hr)/s In SI, the units of average acceleration are m/s (meters per second) divided by s, giving us (m/s)/s (meters per second per second). If, for example, your velocity changes from 15 m/s to 35 m/s as your stopwatch advances from 3 s to 7 s, your velocity increases on the average by 5 m/s during each second of the 4-s time interval. Then

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{20\ \frac{\text{m}}{\text{s}}}{4\ \text{s}} = \frac{5\ \frac{\text{m}}{\text{s}}}{\text{s}} \quad \text{or} \quad 5\left(\frac{\text{m}}{\text{s}}\right)/\text{s}$$

Note that mathematically, (m/s)/s means

$$\frac{\frac{\text{m}}{\text{s}}}{\text{s}} = \frac{\frac{\text{m}}{\text{s}}}{\frac{\text{s}}{1}} = \frac{\frac{\text{m}}{\text{s}} \times \frac{1}{\text{s}}}{\frac{\text{s}}{1} \times \frac{1}{\text{s}}} = \frac{\frac{\text{m}}{\text{s} \times \text{s}}}{1} = \frac{\text{m}}{\text{s}^2}$$

so you will also see (m/s)/s written as m/s^2 (meters per second squared) or as ms^{-2}. No matter—it still means the same thing.

✦ Average Acceleration as Slope of *v* Versus *t* Graph

If we apply the general definition of slope

$$\text{slope} = \frac{\Delta(\textit{quantity plotted vertically})}{\Delta(\textit{quantity plotted horizontally})}$$

to v versus t plots such as those in Figure 2-12, we find that the slope is $\frac{\Delta v}{\Delta t}$. But $\frac{\Delta v}{\Delta t}$ is our definition of average acceleration.

The **slope of a *v* versus *t* graph** between t_1 and t_2 is equal to the **average acceleration** over that interval.

If the graph is not a straight line between t_1 and t_2, then the slope is understood to mean the slope of the secant connecting

(t_1, v_1) and (t_2, v_2).

The graph in Figure 2-12*a* rises left to right—its slope is always positive. The descending graph in Figure 2-12*b* always has a negative slope. But the slope is the average acceleration. So as we concluded previously, the acceleration is always positive for the motion graphed in Figure 2-12*a* and always negative for the motion graphed in Figure 2-12*b*.

✦ **Instantaneous Acceleration** Like average velocity, average acceleration is defined over an interval. As we did for average velocity in WebLink 2-1, we can find average accelerations for progressively smaller time intervals. The value approached as the interval closes in on a particular instant is defined to be the acceleration at that instant.

Definition: In one dimension,

$$\textit{instantaneous acceleration} \quad a \equiv \lim_{\Delta t \to 0} \bar{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \qquad (2\text{-}8)$$

Without a bar, *a* denotes instantaneous acceleration.

When the *v* versus *t* graph is not a straight line, we can "zoom in" on a segment that includes any instant *t* that interests us. If we zoom in close enough, the segment is essentially straight, and its slope gives the instantaneous acceleration at that instant.

Example 2-5 *Determining Instantaneous Accelerations by Finding Slopes*

The motion of a certain object is represented by the *v* versus *t* graph in Figure 2-14. Find approximate values of the instantaneous acceleration

a. at $t = 2.0$ s
b. at $t = 20.0$ s

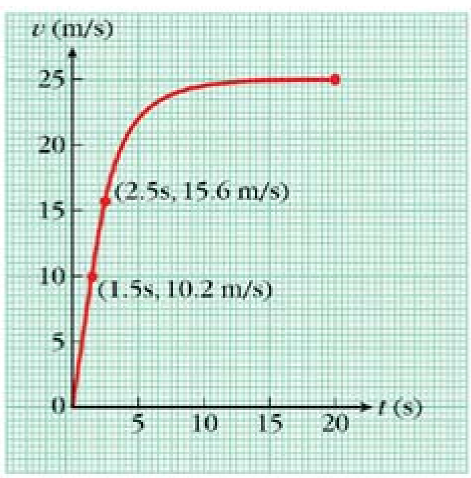

Figure 2-14 **Graph of *v* versus *t* for Example 2-5.**

Solution

Choice of approach. Pick a small segment of the graph that includes each instant, then find the slope of each segment.

The mathematical solution.

a. To include $t = 2.0$ s, we arbitrarily pick the interval from $t_1 = 1.5$ s to $t_2 = 2.5$ s. (You can redo this choosing an even smaller interval to see how much it changes your result—see Problem 2-100 at the end of the chapter.) From the graph, we find that the velocity at $t_1 = 1.5$ s has the value $v_1 = 10.2$ m/s, and at $t_2 = 2.5$ s, its value is $v_2 = 15.6$ m/s. The slope of the chosen segment is

$$\frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{15.6\text{ m/s} - 10.2\text{ m/s}}{2.5\text{ s} - 1.5\text{ s}} = 5.4\text{ (m/s) / s}$$

Then at $t = 2.0$ s, the instantaneous acceleration a $\approx$ **5.4 (m/s)/s** or **5.4 m/s^2** .

b. As we go beyond $t = 10$ s, we see the graph becoming more nearly horizontal. Because the slope of any horizontal line is zero, the slope of a small enough segment including $t = 20.0$ s is approximately zero, and so a $\approx$ **0**.

Making sense of the results. The value obtained in **b** reflects the fact that by $t = 20$ s, the velocity is no longer changing.

- **Related homework: Problems 2-28, 2-29, and 2-100.**