

2-2 A Vocabulary for Describing Motion

We often use terms like *speed*, *distance*, and *time* when describing motion. But even these terms are not sufficiently clear for our purposes. In ordinary language, for example, we use the word *time* in two distinct ways. If I ask, “What time is it?” you might reply “a quarter to two” or “9:54 AM.” If I ask, “How much time do you need?” you might respond with something like “2 hours.” No proper response to the first question legitimately answers the second and vice versa; yet in both cases I am asking for “the time.” We need to distinguish between specific **instants** or *clock readings*, on the one hand, and **time intervals** or *durations* on the other.



For some of nature's creatures, one-dimensional motion comes naturally.

◆ **Instants and Intervals** If I ask you for an *instant* (“At what time ...?”), you need look at your watch only once to respond. If I ask you for a time interval (“How long does it take to do X?”), you must look at your watch twice, when you start and when you finish. If your readings were 3 o'clock and 5 o'clock, you would then conclude that it took you $5 \text{ hr} - 3 \text{ hr} = 2 \text{ hr}$. We will denote an **instant**, a *single clock or watch reading*, by t . To find an **interval**, which we denote by Δt , we need to obtain two readings, t_1 and t_2 , and subtract the earlier reading from the later one:

$$\Delta t \equiv t_2 - t_1$$

Interval \equiv difference between two **instants** .

The triple-bar equal sign tells you this is a definition.

◆ **Point Objects** When an object moves, it changes its position during some time interval. If we are not careful, however, this claim can get us into some silly discussions, such as:

You: But I can wave my arms around while standing in one place.

I: But then your arms are changing *their* positions.

You: Yes, but my hands are moving further than my biceps. Which change in position are we talking about?

etc.

We avoid all this by thinking first about an object small enough so that we can treat it mentally as though it were no more than a point. We call such an object a **point object** or **point particle**.

There is no such thing as a point object in nature. But it can be a useful way of thinking about a larger object when the object's size doesn't matter. For example, in describing the motion of a car on a cross-country drive, the length of your car is such a tiny fraction of the distance traveled that it might as well be a point. Physicists say its size is *negligible* in this situation. But if your car goes into a skid on an icy turn, you are very concerned that the tail of the car may be moving

differently than the front end; in that case, a point object approximation would cut out much of what is of interest in analyzing the situation. We could, however, treat each tiny piece of the car as a separate point object. In that way, developing a description of the motion of point objects lays a foundation for describing the motions of all objects. We will start by describing one-dimensional (straight-line) motion. We'll then extend our ideas to two-dimensional motion (Figure 2-1) in Chapter 3.

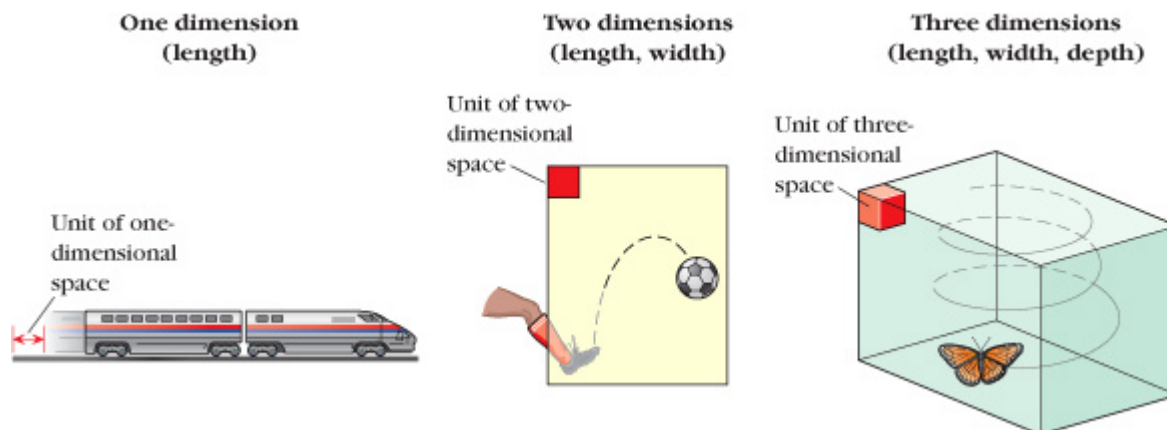


Figure 2-1 Motion in one, two, and three dimensions.

◆ **Positions and Distances** We can describe the **position** of any point or point object by giving its coordinates on some set of coordinate axes. Each axis is basically a real number line. For one-dimensional motion, position means location along a single real number line, on which consecutive integers are one unit (such as a meter) apart.

We can picture such a number line stretched along a long, straight road (Figure 2-2). But where should the zero go? The choice is yours; it depends on what question you wish to address. If you are interested in how far things are from Newtonville in Figure 2-2, you would choose the red number line, with its zero fixed at Newtonville. If you care about distances from Keplertown, on the other hand, the blue number line's zero is better located. Once we choose a particular number line, we can measure an object's position at various instants during its motion. In Figure 2-3a, for example, the ball's position has been measured at $t = 2$ s and again at $t = 5$ s. **STOP&Think** Based on these two measurements, what can we say about the total distance the ball traveled during that 3-s interval? ◆

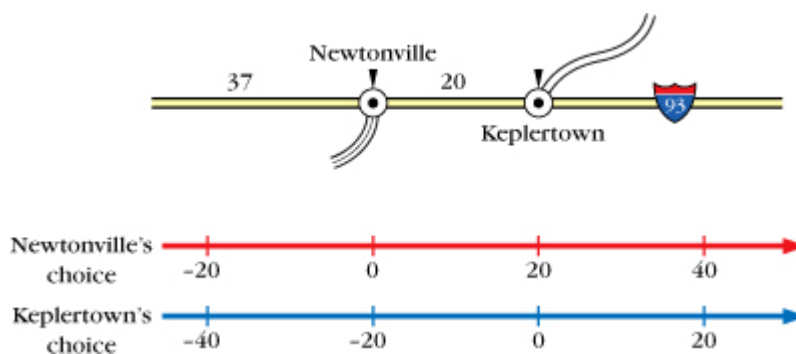


Figure 2-2 Number lines or axes to indicate position.

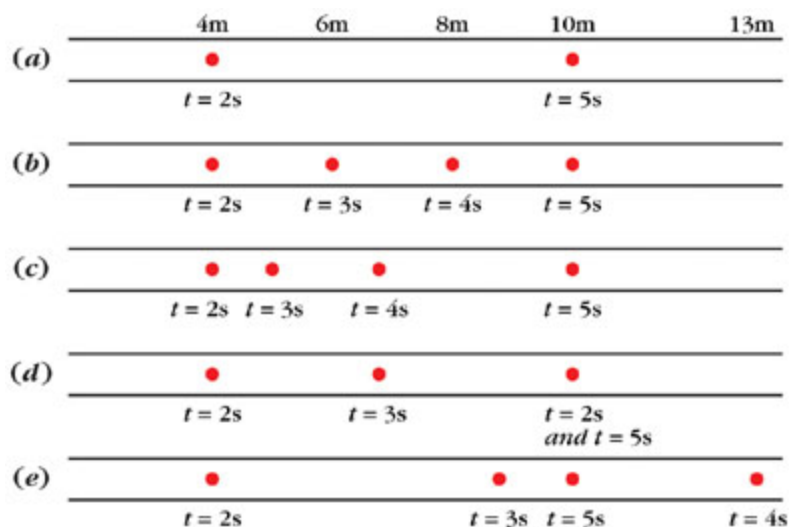


Figure 2-3 Different ways of achieving the same displacement and average velocity.

In fact, those two measurements provide no details about how the ball moved during the interval. Any of the scenarios shown (Figures 2-3b to 2-3e) is possible; the number of possibilities is infinite. If (as in Figure 2-3e) the motion is not all in one direction, the total distance traveled is *not* just $10\text{ m} - 4\text{ m} = 6\text{ m}$. Even in Figure 2-3b, we cannot say for certain that the ball traveled 6 m, because there might have been back-and-forth motion *in between* clock readings (say, between $t = 3\text{ s}$ and $t = 4\text{ s}$). All we know for sure is that in each of the scenarios the *net* or *resulting* change in position was 6 m to the right during the 3-s interval depicted. We call this change in position the **displacement** Δx in one dimension:

Definition: In one dimension,

$$\text{displacement } \Delta x \equiv x_2 - x_1 \quad (2-1)$$

- The displacement is a difference between two positions in space, just as an interval is a difference between two instants in time. It can have both positive and negative values, as you will see in Example 2-1.
- If an object ends up at x_2 and if x_1 is the zero point or origin, then $\Delta x = x_2$; in other words, an object's position is its displacement from the origin.

The displacement tells you how far x_2 is from x_1 , but not how far an object has traveled to get to x_2 from x_1 . In contrast, distance is the length of the actual path traveled. Each piece of the overall length is positive, so the longer you travel, the greater the distance you've gone.

For each straight line path segment where there is no reversal of travel direction,

$$\text{distance} = |\Delta x| = |x_2 - x_1| \quad (2-2)$$

$|\Delta x|$ denotes “absolute value of Δx ”; it is always positive.

Table 2-1 summarizes the relationships among these quantities and the type of question each quantity addresses.

Table 2-1 Basic Quantities Used in Describing Motion

Term	Position	Displacement	Instant	Time Interval
Meaning	Number line or coordinate	Space interval	Clock reading,	Time duration, time

	axis reading, point in space		point in time	elapsed
Symbol	x	$\Delta x = x_2 - x_1$	t	$\Delta t = t_2 - t_1$
Question addressed	Where ... ?	How far ... ? (and sign tells, Which way ... ?)	When ... ? At what time ... ?	How long ... ? How much time ... ?

◆ **Notation** A straight line path may be horizontal, diagonal, or vertical (Figure 2-4). If we use y instead of x for position along a vertical path, then the displacement between two points would be $\Delta y = y_2 - y_1$.

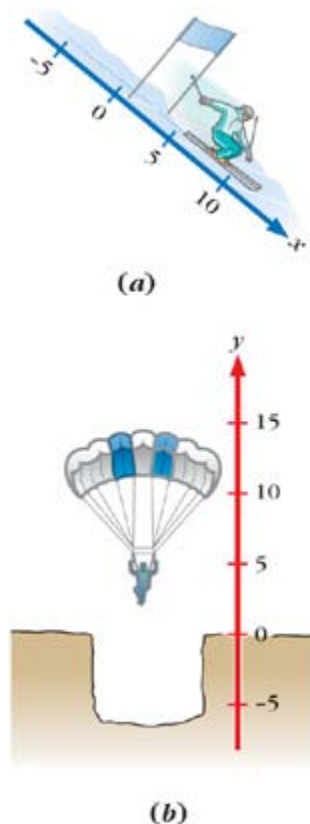


Figure 2-4 Axes indicating position needn't be horizontal.

◆ **Negative Values** When we use a real number line to identify position along a straight line, half of all possible positions will have negative values. In Figure 2-2, a position 3 m to the left of the origin is expressed as $x = -3$ m.

Example 2-1 *Negative and Positive Displacements*

Find the displacement of a cyclist who rides

- from a position of 500 m to a position of 300 m.
- from a position of -500 m to a position of -300 m.
- Find the distance traveled by the cyclist in each case (assuming no backtracking).

Solution

Sketch the situation. It is important to have a clear picture of what is happening. Figure 2-5 shows the displacements asked in **a** and **b**.

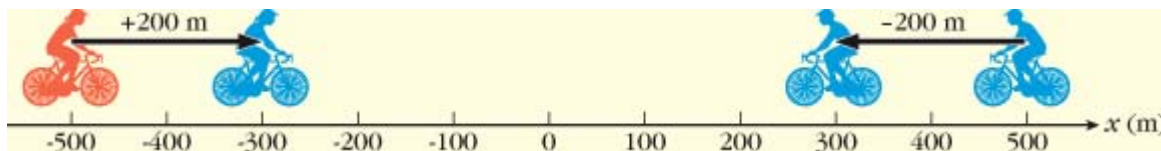


Figure 2-5 Positions and displacements for Example 2-1.

Use the definition. Displacement is defined as $\Delta x = x_2 - x_1$, where x_1 is the earlier and x_2 the later position. In **a**, $x_1 = 500$ m = 300 m, so

$$\Delta x = 300 \text{ m} - 500 \text{ m} = -200 \text{ m}$$

and is negative. In **b**, however, $x_1 = -500$ m and $x_2 = -300$ m, so

$$\Delta x = -300 \text{ m} - (-500 \text{ m}) = -300 \text{ m} + 500 \text{ m} = +200 \text{ m}$$

and is positive.

c. The absolute value of either displacement is 200 m, so the *distance* traveled is 200 m in each case. Distances are always positive.

Making sense of the results. In all cases,
if the motion is to the right along a left-to-right real number line, the displacement is positive and if it is to the left, the displacement is negative.

Figure 2-5 illustrates this point.

◆ **Related homework: Problems 2-7, 2-8, and 2-9.**

In one sense, the displacement provides more information than the distance: It answers not only “How far?” but “Which way?” as well. In the same spirit, we wish to address “How fast?” and “Which way?” together. To do so, we define a quantity \bar{v} called the *average velocity during a time interval Δt* (the bar over the v denotes *average*):

Definition: During the time interval Δt , the

$$\text{average velocity } \bar{v} \equiv \frac{\text{displacement}}{\text{time interval}} \equiv \frac{\Delta x}{\Delta t} \equiv \frac{x_2 - x_1}{t_2 - t_1} \quad (2-3)$$

You can find the average velocity of an object using a meter stick and a clock. The definition is equivalent to the following procedure:

Procedure 2-1

Determining Average Velocity in One Dimension

1. Measure the object's coordinate x_1 at instant t_1 .
2. Measure the object's coordinate x_2 at instant t_2 (later than t_1).
3. Calculate the increments $\Delta x = x_2 - x_1$ and $\Delta t = t_2 - t_1$.

4. Divide Δx by Δt to obtain \bar{v} .

With t_2 as the later instant, Δt is positive. Then \bar{v} has the same sign as the displacement Δx , and thus is in the same direction. We still need to clarify two aspects of average velocity: (1) why *average*? and (2) how is velocity different than our everyday notion of speed?

◆ **Why Average?** Consider Figure 2-3. The ball in Figure 2-3a is at position $x_1 = 4$ m when the clock reads $t_1 = 2$ s and at $x_2 = 10$ m when $t_2 = 5$ s. The ball's displacement is therefore $\Delta x = 10$ m $-$ 4 m = +6 m (to the right) during a time interval $\Delta t = 5$ s $-$ 2 s = 3 s. During this time interval, the average velocity is $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{+6 \text{ m}}{3 \text{ s}} = +2 \text{ m/s}$ (also to the right). But Figure 2-3a provides no information about how the ball moved between the two instants $t = 2$ s and $t = 5$ s. The possible scenarios in Figures 2-3b to 2-3e show that the ball may have traveled at uniform speed, sped up, gotten to $x = 10$ m quickly and then stopped there until $t = 5$ s, and so on. We can't say which scenario the ball followed or how fast it was going at any particular instant during the interval, only that its velocity averaged 2 m/s for the whole 3-s interval.

◆ **How is Average Velocity Different than Speed (or Average Speed)?** Physicists are basically stating the everyday notion that most people have of speed when they define *average speed* as the total *distance* traveled divided by the time spent traveling:

$$\text{average speed} \equiv \frac{\text{total distance traveled}}{\text{total time interval}} \quad (2-4)$$



An albatross gliding on prevailing winds. In the Southern Hemisphere, albatrosses have been known to go completely around the Earth in this way. At 40° south latitude, what information would you need to look up to calculate how long this trip would take?

For each segment with no direction reversals, the key difference is this: *The average velocity can be positive or negative; the sign indicates its direction. The average speed is always positive; it provides no information about direction.* When there are direction reversals, the average speed and average velocity can have numerically different values, as in the following example.

Example 2-2 Average Speed versus Average Velocity

For the motion of the ball in Figure 2-3e during the time interval from $t = 2$ s to $t = 5$ s, find **a.** the total distance, **b.** the total displacement, **c.** the average speed, and **d.** the average velocity. Assume the only change in direction occurs at $t = 4$ s.

Solution

Choice of approach. We can find the average speed and average velocity from their definitions (Equations 2-4 and 2-3). Be

distance equals $|x_2 - x_1|$ only for path segments in which the object doesn't reverse direction, we must find separately the distance traveled from $t = 2$ s to $t = 4$ s and from $t = 4$ s to $t = 5$ s, and then add the two distances. We will get the same displacement however, whether we add the displacements for the two shorter intervals or just subtract the position at $t = 2$ s from the position at $t = 5$ s.

The mathematical solution. To make comparison easier, we will arrange the calculations in a table.

Distance			Displacement Δx			Average Speed	Average Velocity
Between $t = 2$ s and $t = 4$ s	Between $t = 4$ s and $t = 5$ s	Total	Between $t = 2$ s and $t = 4$ s	Between $t = 4$ s and $t = 5$ s	Total	Between $t = 2$ s and $t = 5$ s	$\left(\frac{\text{total distance}}{\Delta t} \right)$
$ 13 \text{ m} - 4 \text{ m} = 9 \text{ m}$	$ 10 \text{ m} - 13 \text{ m} = 3 \text{ m}$	$9 \text{ m} + 3 \text{ m} = 12 \text{ m}$	$13 \text{ m} - 4 \text{ m} = 9 \text{ m}$	$10 \text{ m} - 13 \text{ m} = -3 \text{ m}$	$9 \text{ m} + (-3 \text{ m}) = 6 \text{ m}$	$10 \text{ m} - 4 \text{ m} = 6 \text{ m}$	$\frac{12 \text{ m}}{3 \text{ s}} = 4 \text{ m/s}$
							$\frac{6 \text{ m}}{3 \text{ s}} = 2 \text{ m/s}$

A critical feature of the calculations is that unlike the distance, the displacement from $t = 4$ s to $t = 5$ s is negative. This leads to an average velocity that is different than the average speed.

◆ **Related homework: Problems 2-10, 2-11, and 2-12.**

◆ Instantaneous Velocity



For **WebLink 2-1: A Spaceship in a Resistive Medium**, go to www.wiley.com/college/touger

To find an average velocity experimentally, you must take position readings at two instants. These measurements cannot tell you how fast something is going at any *single* instant. If you want to know the velocity at a particular instant t , you must approach it by taking readings for smaller and smaller time intervals containing that instant. Using these measurements, you find the average velocity for each interval. These average velocity values close in on a particular value (called a *limiting value*) as Δt shrinks closer and closer to zero ($\Delta t \rightarrow 0$). To see in detail how this works, work through WebLink 2-1. This limiting value is what we call the *instantaneous* velocity at t .

Definition: In one dimension,

$$\text{instantaneous velocity } v \equiv \lim_{\Delta t \rightarrow 0} \bar{v} \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} \quad (2-5)$$

The symbol v with no bar above it denotes *instantaneous velocity*. Just as average velocity only has meaning over a particular time interval, instantaneous velocity only has meaning at a particular point in time—a particular *instant* t .

- When the time interval containing an instant is reduced enough to exclude all direction changes, distance = $|\Delta x|$ and average speed = $\frac{|\Delta x|}{\Delta t}$. Then, shrinking Δt to zero, we can take the absolute value of the instantaneous velocity to be the *instantaneous speed*.
- If an object does reverse direction, in slow motion it would seem to stop and then begin moving in the opposite direction. There is an instant of reversal. If the velocity is positive before this instant, then after, it is negative. In changing sign, it

must pass through zero. At this instant, then, the instantaneous velocity is zero.

We've defined the average velocity as $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$. Suppose we choose the first instant t_1 to be $t = 0$. This is

like resetting our stopwatch to $t = 0$ when we start our observations. We can let x_0 denote the object's position at this instant, so that x_0 and 0 become the "values" of x_1 and t_1 . If x represents its position at any later instant t (so that these become the "values" of x_2 and t_2), then

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t - 0} = \frac{x - x_0}{t}$$

Solving for x in this equation gives us

$$x = x_0 + \bar{v} t \quad (2-6)$$

In words,

$$\textit{position at time } t = \textit{initial position} + \textit{additional displacement traveled} \\ \textit{during the interval } \Delta t = t - 0$$

Both x_0 and $\bar{v} t$ are lengths. Adding them gives a total length.

✦ **Uniform Motion** When an object's velocity is **uniform** (the same at every instant during the time interval being analyzed), we do not have to distinguish between average velocity \bar{v} and instantaneous velocity v . Then Equation 2-6 becomes

$$x = x_0 + vt \quad (\text{uniform motion only}) \quad (2-6a)$$