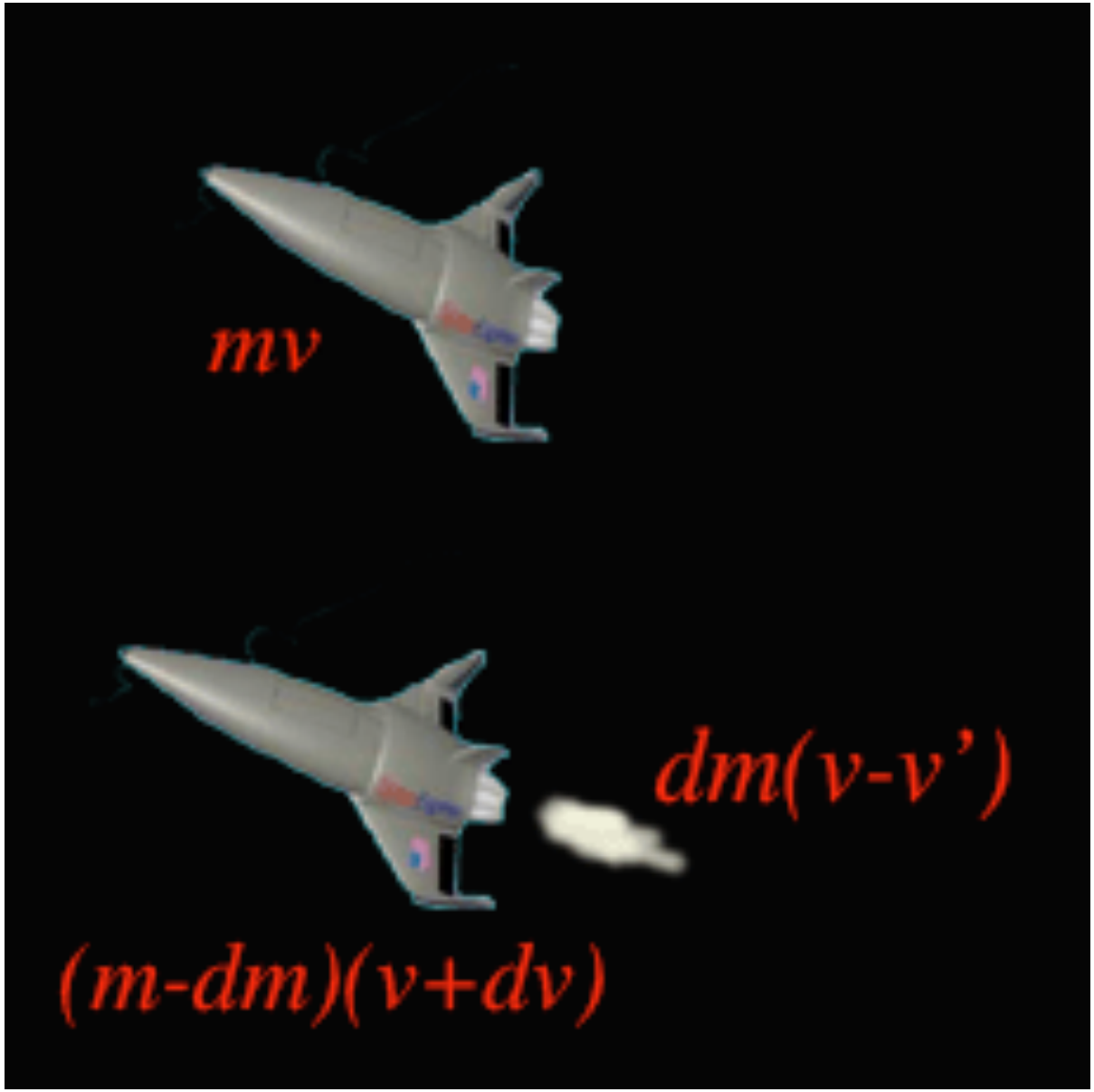

$$mv = (m - dm)(v + dv) + dm(v - v')$$

$$mv = mv - vdm + mdv - dmdv + vdm - v'dm$$

$$mv = mv - vdm + mdv - dmdv + vdm - v'dm$$

$$0 = mdv - v'dm$$

$$\int_{v_0}^{v_f} dv = v' \int_{m_0}^{m} \frac{dm}{m}$$

$$v\Big|_{v_0}^{v_f} = v' \ln m\Big|_{m_0}^{m_f}$$

$$v_f - v_0 = v'\left(\ln m_f - \ln m_0\right) = v' \ln\left(\frac{m_f}{m_0}\right)$$

$$v = v' \ln\left(\frac{m}{m_0}\right)$$