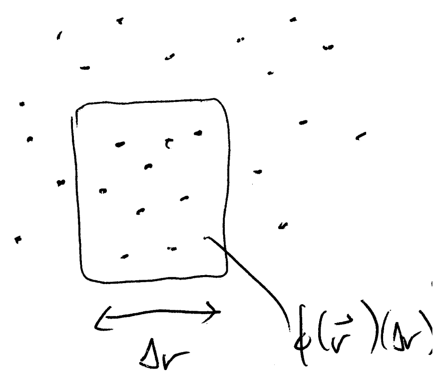
* Even if all fundamental laws and the complete zoology of particles are <u>known</u>, we can still only solve for a limited set of single- and few-body behaviours
  → only a handful of 3-body problems are integrable!
  → related to the notion of computational irreducibility

* What to do with 10^{27} interacting particles (a litre of water, say)?
  → formulate coarse-grained theories in terms of macroscopic variables

  e.g. average local values of particle or momentum densities, magnetization; fluctuations of such quantities or their response to external fields



$l_{micro} \ll \Delta r \ll L_{macro}$

  → observe and characterize the many different thermodynamically stable phases of matter

  e.g. fluids flow
  solids are rigid
  Some matter is transparent ... others coloured
  transport of heat and charge in insulators, metals and semiconductors

* CMP provides a <u>framework</u> for understanding the properties of various phases of matter

  ① microscopic picture: large group of particles interacting via well-known (mostly Coulombic) forces

② focus on macroscopic properties: rather than trajectories of individual particles, we use a local averaging

→ statistical mechanics + thermodynamics

→ macroscopic variables that vary slowly and continuously in space (classical + quantum continuum field theories)

③ important connection to the <u>organization</u> of matter: geometric properties, patterns and regularity (or the lack of it)

e.g. regular solid

$$|\psi\rangle = |\cdots\rangle$$

ordered arrangement

liquid

$$|\psi\rangle = \sum_{\text{all configs}} |\cdots\rangle$$

many nearly equivalent low-energy configurations ⇒ almost no cost to deformation, hence a liquid has <u>no shape</u>

glass

$$|\psi\rangle = |\cdots\rangle$$

disordered but "frozen"

④ unifying concepts: conservation laws (conserved quantities = constants of the motion)

and broken symmetries

e.g. in an isolated system, particle number, energy, and momentum are conserved.

$\boxed{\text{the system at sufficiently high temperature}}$

→ necessarily disordered, uncorrelated, homogeneous and isotropic

→ has full rotational and translational symmetry of free space

→ low-freq. long-wavelength behavior controlled by hydrodynamical eqns

via cons. mass $\quad \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$

via cons. momentum $\quad \rho\left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v}\right) = \vec{\nabla} \cdot \vec{\sigma}$

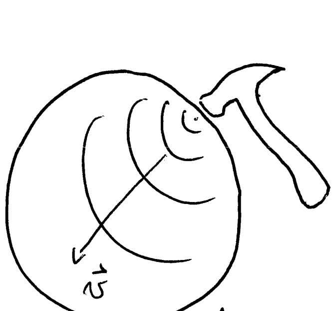
some local stress tensor

$\boxed{\text{System at low temperature}}$

→ new thermodynamically stable phases condense, having progressively <u>lower</u> symmetry

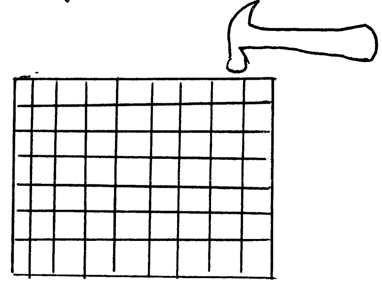
e.g. a periodic crystal is invariant wrt a discrete set of spatial transformations

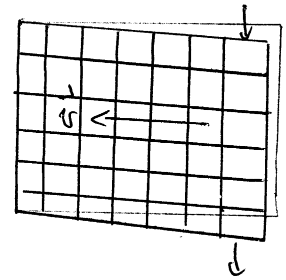
→ associated with each broken symmetry are distortions, defects, and dynamical modes (which provide a pathway to restoring the high-symmetry state) e.g. bag of liquid



response to hammer blow is a longitudinal compression wave

crystalline solid

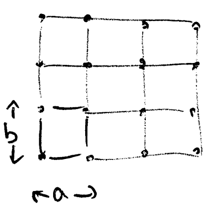




shear response = sound wave mode that doesn't exist in the liquid!

Order parameter theory of a crystal distortion

* Suppose atoms in a solid feel a potential that stabilizes a cubic crystal structure; along a cleaved edge we see a square lattice with dimensions $a=b$

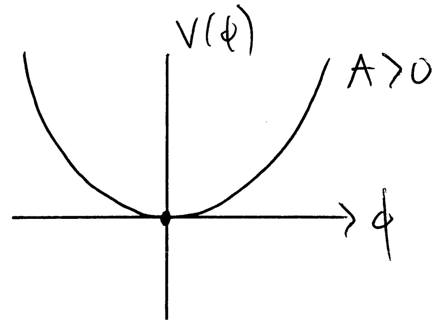


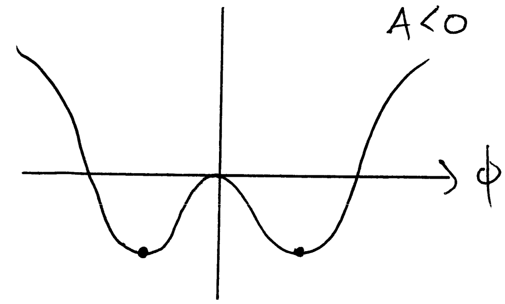
→ invariance under $90°$ rotations means that the overall free energy must be invariant under the swap $(a,b)\to(b,a)$

→ Hence $V(a,b) = V(b,a)$ or in an alternative set of coordinates $V(ab, \frac{a}{b}) = V(\phi)$ where $\phi = \lg \frac{a}{b}$ and $V(\phi)$ is even is ϕ (since $\phi \to -\phi$ equals $(a,b)\to(b,a)$)

→ for small distortions, expand in powerseries

$$V(\phi) = A\phi^2 + B\phi^4 \quad (B>0 \text{ for stability})$$

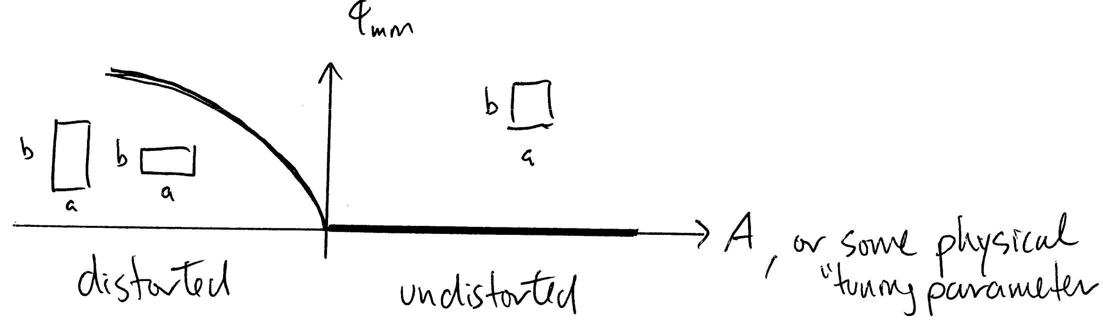




$$\left.\begin{aligned} V'(\phi) &= 2A\phi + 4B\phi^3 \\ &= 2\phi(A + 2B\phi^2) = 0 \end{aligned}\right\} \text{ stationary when } \phi = 0 \text{ or } \phi = \pm\sqrt{\frac{-A}{2B}}$$

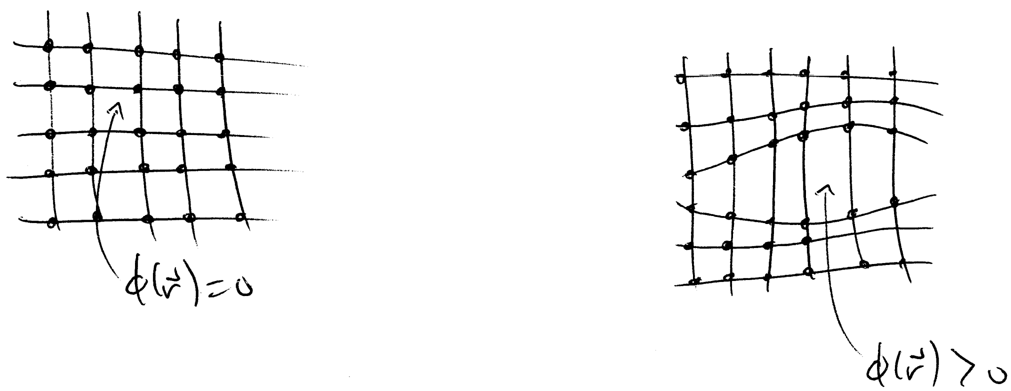
$$\text{Hence } (\phi, V)_{min} = \begin{cases} (0, 0) & \text{for } A > 0 \\ \left(\pm\sqrt{\frac{-A}{2B}}, \frac{-A^2}{4B}\right) & \text{for } A < 0 \end{cases}$$

* continuous symmetry breaking as A is tuned through zero



→ A may have some dependence on physical quantities such as pressure or temperature

* local stability determined by a field $\phi(\vec{r})$, which serves as a position-dependent order parameter

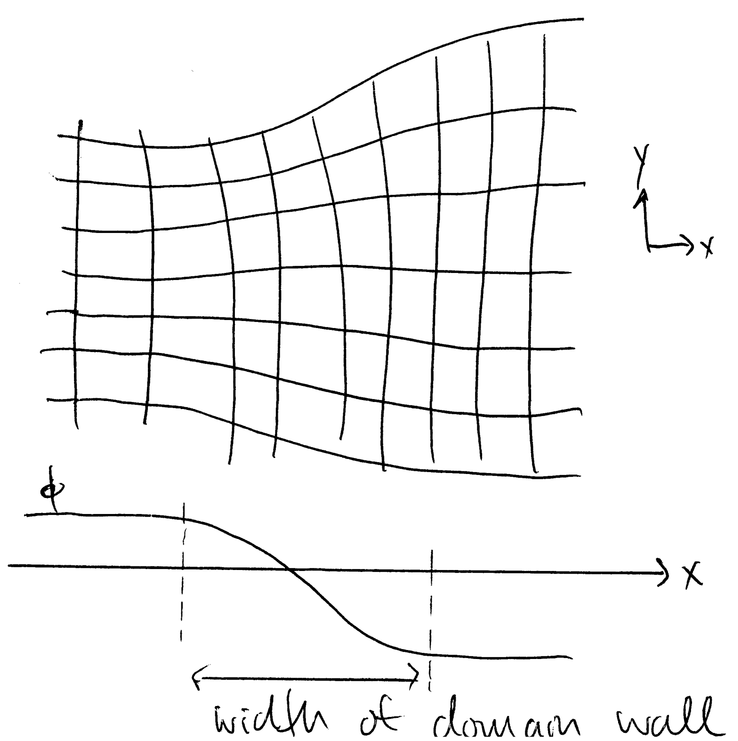


with an integrated free energy

$$F[\phi] = \int d^2r \left(\underbrace{\frac{\rho}{2}(\nabla\phi)^2}_{} + V(\phi) \right)$$

functional of $\phi(\vec{r})$

encodes the elastic energy cost of mismatches between horizontally and vertically deformed domains; ρ is the stiffness of deformation



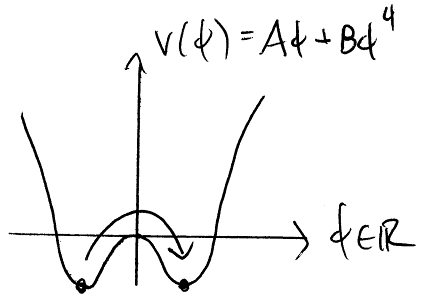
width of domain wall / grain boundary

* Diffusive dynamics (roughly) given by

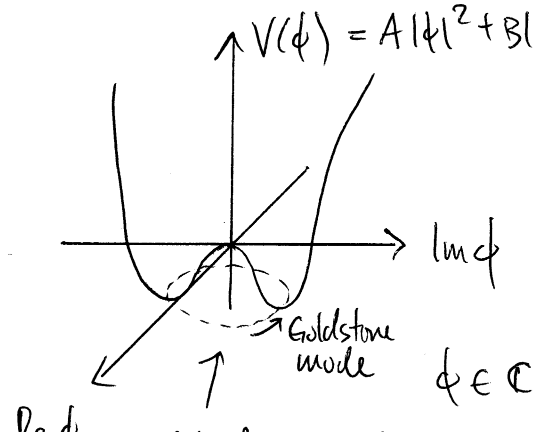
$$\frac{\partial\phi}{\partial t} \sim -\frac{F[\phi]}{\delta\phi} = \rho_s \nabla^2\phi + 2A\phi - 4B\phi^3$$

→ simple model of competition between two competing phases

* More possibilities when the order parameter is not a scalar (e.g. complex number or a vector)



two ground states connected by barrier tunnelling



U(1) degenerate family of ground states