Scientific Computing: Lecture 26

- Time series data
- Fourier Analysis
- Discrete Fourier Transforms
- Python Tools

CLASS NOTES

- Last Class!
- WORK ON PROJECTS!

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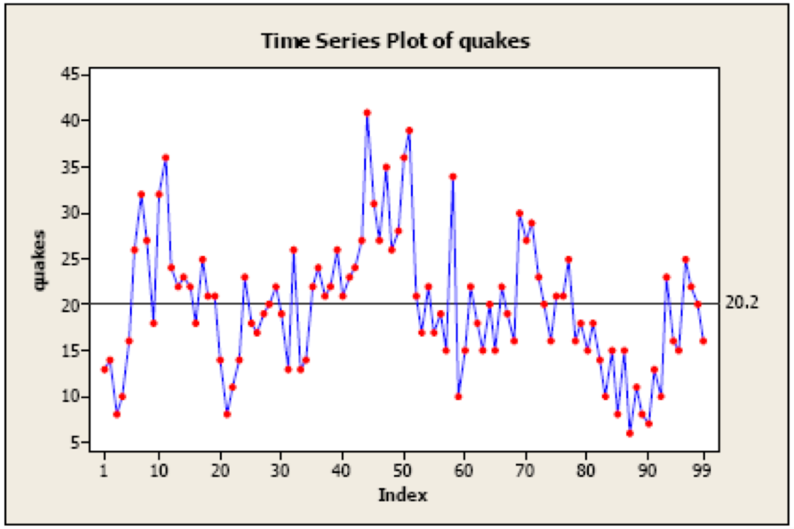
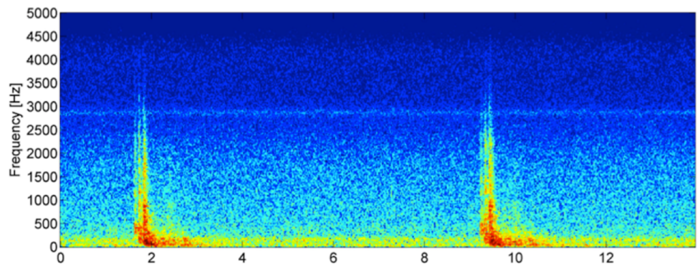
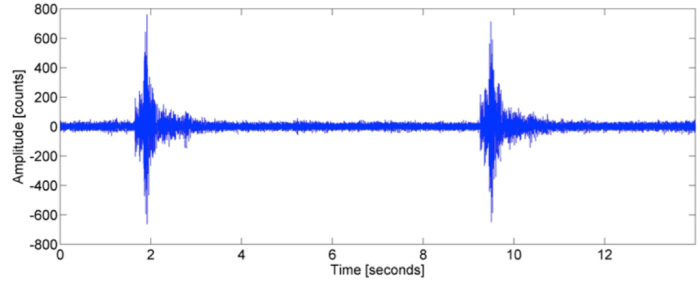
Time Series Analysis

- Analysis of time series data is important in many areas of science and engineering
  - Acoustics
  - Optics
  - Geophysics
  - Health
  - Biological systems
  - Wireless communications







Fourier Transform

- A Fourier transform converts time series data (a response in time) into frequency data (a response in frequency space).
- It can also be used on real space data to convert it to wavenumber space (sometimes called “k” space from quantum mechanics or crystallography)
- For a continuous time series signal $f(t)$, the Fourier transform is:

$$\hat{f}(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-2\pi i \omega t} dt$$

- This results in a complex function in frequency space.

Discrete Fourier Transform

- DFT can be performed on discrete data sets rather than continuous functions
  - Integrals become summations
- f_k is the signal measured at time step *k* with a total of N samples.

$$F_n = \sum_{k=1}^{N} f_k e^{-2\pi i k n/N}$$

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Fourier Series

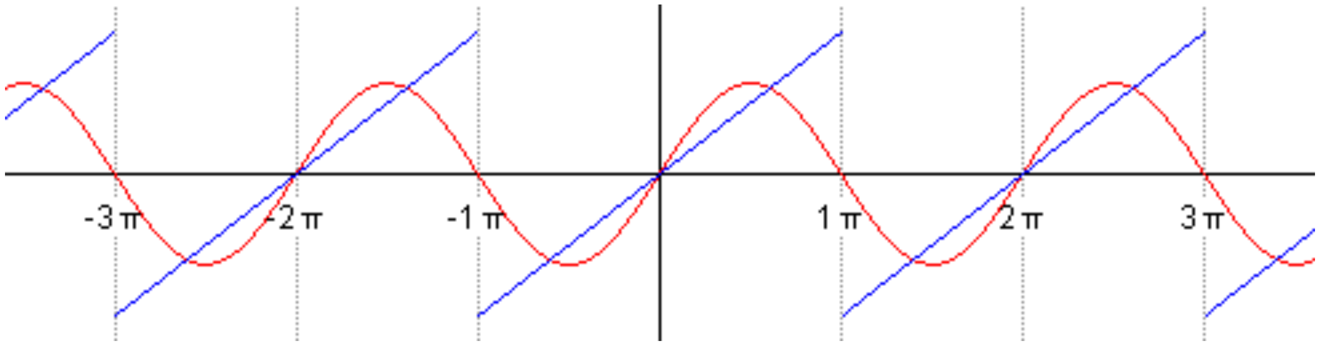
- Basic concept is that any function can be represented by an infinite series of harmonic sine and cosine functions.
- For a function *f(x)* that is defined over a domain $[-\pi, \pi]$

$$f(x) = A_0/2 + \sum_{n=1}^{N} [A_n \cos(nx) + B_n \sin(nx)]$$

$$A_0 = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)dx \quad A_n = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cos(nx)dx$$

$$B_n = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\sin(nx)dx$$

Sawtooth and Square Waves



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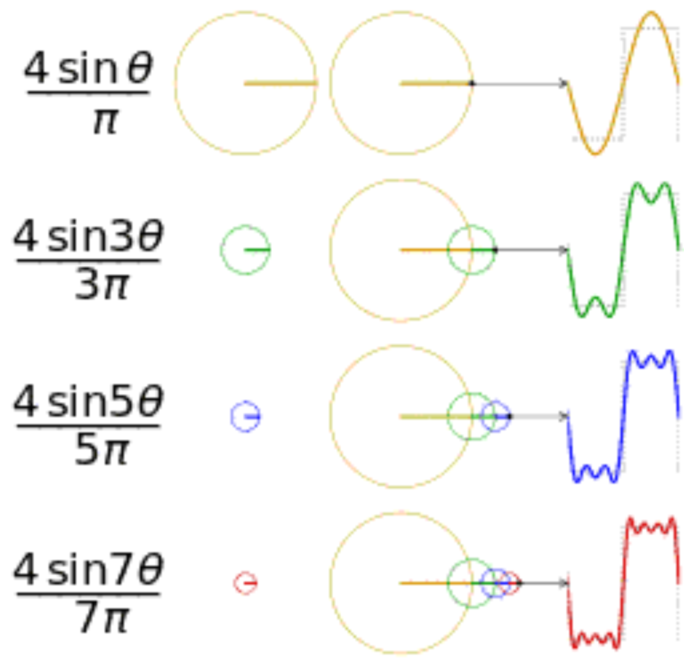


Importance of Sampling Rates

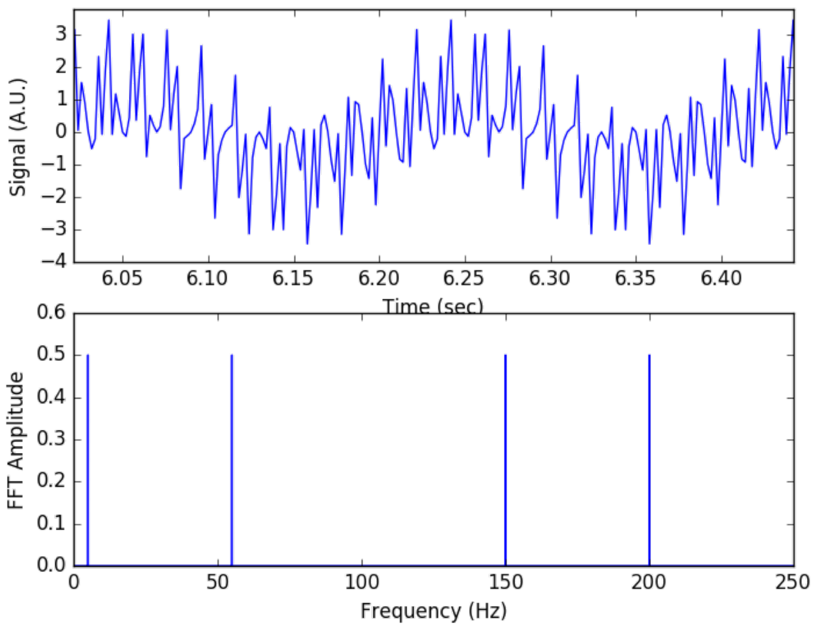
- Nyquist Frequency
  - For discrete data, there is a minimum time step between data points. This means there is a maximum frequency that can be determined:
  - $N_f = \frac{1}{2\,\Delta t}$ although better to limit yourself to a maximum frequency that is about N_f / 10.

Fast Fourier Transform

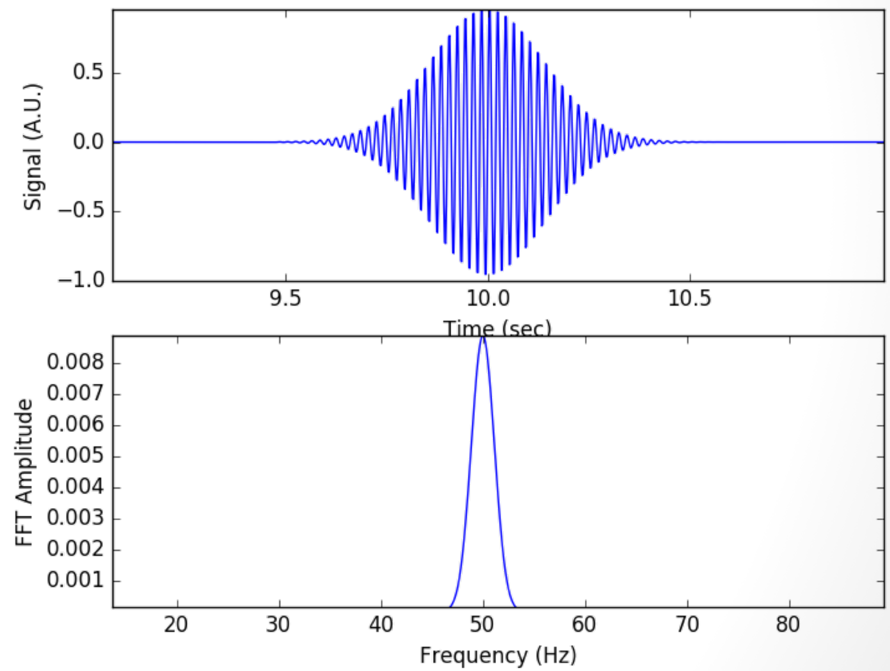
- DFT requires $2N^2$ computations which is rather expensive.
- Cooley and Tukey came up a short cut that reduced the number of computations to 2 N log(N), but a restriction was that N was a power of 2 (256, 512, 1028, …)
- Most modern FFT tools actually use a modification of the original FFT which can work with any number of sample points – although powers of 2 are most efficient.

Examples

- Multi-tone (harmonic)

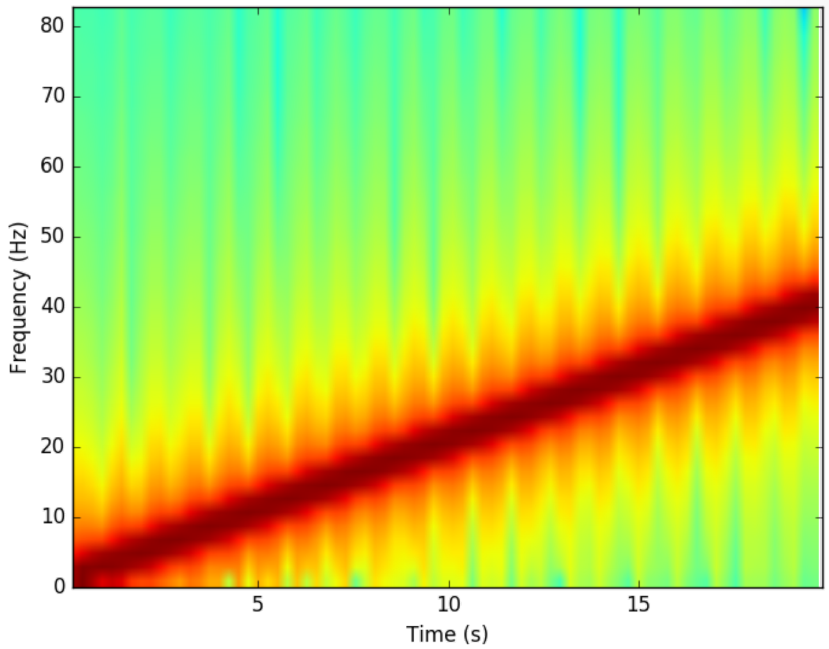


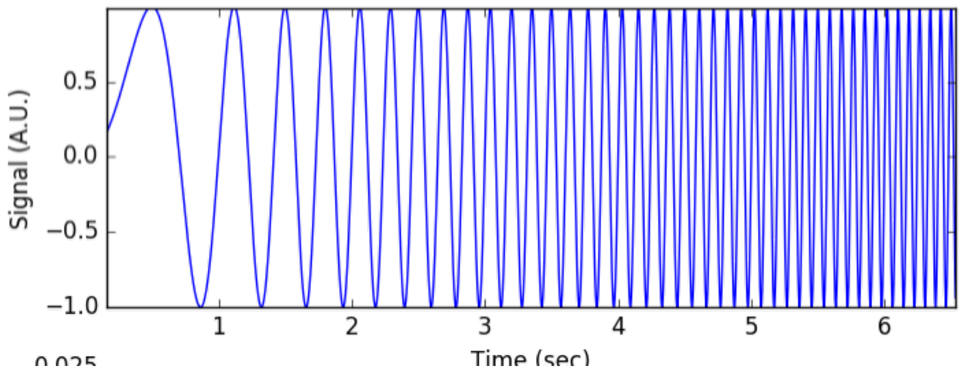
Wavepacket



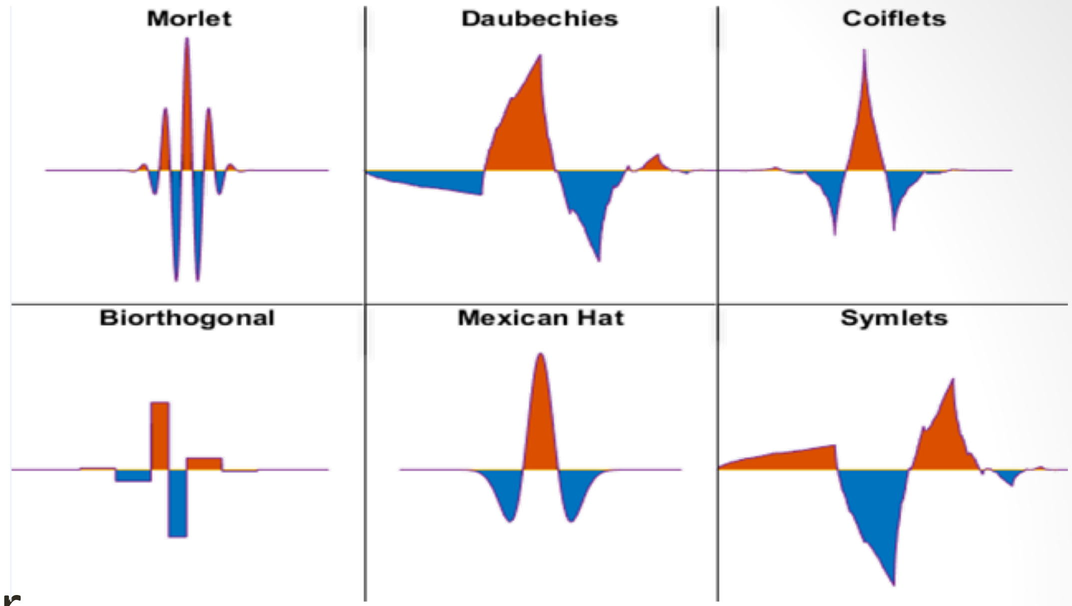
Spectrogram

- Often the frequency content in a signal varies with time.
- A spectrogram performs an FFT on a running window in the signal.
- Here is a chirp signal





Other tools: Wavelets



- A powerful new method for time series analysis is wavelets.
- Wavelets do a better job of analyzing signals with sharp jumps
- Instead of sines and cosines (which go on forever in time), wavelets are functions more localized in time.
- There are many good resources on the web.
- We won't go into wavelets here, but educate yourself!