Physics 503: Scientific Computing

Homework #7

Topic: Solutions of ODEs: the Lorenz climate model exhibiting Chaos!

Due: Friday 4/6 by midnight- DON'T forget to include output and plots!

Assignment

1. Use the Euler method to solve the system of ODEs

   $$\frac{dx}{dt} = -\sigma x + \sigma y$$

   $$\frac{dy}{dt} = -xz + rx - y$$

   $$\frac{dz}{dt} = xy - bz$$

   for $\sigma = 10.0$, r=29, and b = 8.0/3.0 and starting points: $x_0 = 1.0, y_0 = 1.0, z_0 = 20.1$.
   Make plots of x,y, and z as a function of time. Also create a 3D phase space plot where x,y,z are the variables. (You can use the 3D plotting capabilities of pylab like this:

   ```
   from mpl_toolkits.mplot3d import Axes3D
   fig3d=figure(3)
   ax=Axes3D(fig3d)
   ax.plot3D(x,y,z)
   ```

2. Determine the fixed points for this system of equations. (Pencil and paper or Mathematica)
3. Optional: Given the sensitivity to initial conditions, it would be interesting to compare the Euler method to the solution provided to one produced by odeint in scipy. You can visually compare by plotting both trajectories and numerically by plotting the difference between x,y,z for each solution over time.

A little background

This system was originally developed by Lorenz to model buoyant convections in the atmosphere. Sigma and b are fluid properties of the air and r is the applied temperature gradient. The variables are: x – rate of convective overturning, y & z – horizontal and vertical temperature gradients. He found very interesting mathematical behavior which quickly led to a new field of mathematics called Chaos Theory. One of the hallmarks of chaos is that very small changes in the initial conditions quickly lead to dramatically different time evolution of the variables. The fixed points in this model are called 'strange attractors' because they attract the variables toward them, but the system never settles down into an equilibrium state. You should be able to quickly take code we looked at in class and rework it to solve the Lorenz equations.