1.16

For a potential $V(r) = \frac{\gamma}{r^2}$, derive the differential cross-section.

Try "brute force" method:

$$\phi_m = \ell \int_{\infty}^{r_{min}} \frac{dr}{r^2 \left[2mE - 2mV(r) - \frac{\ell^2}{r^2} \right]^{1/2}} + \pi$$

let $u = \frac{1}{r}$ and $du = -\frac{1}{r^2} dr$

Then

$$\phi_m = -\ell \int_0^{u_{min}} \frac{du}{\left[2mE - (2m\gamma + \ell^2) u^2 \right]^{1/2}} + \pi$$

Now u_{min} when $2mE - (2m\gamma + \ell^2) u^2 = 0$

So $u_{min} = \sqrt{\frac{2mE}{2m\gamma + \ell^2}}$

Then can write ϕ_m as

$$\phi_m = -\int_0^{u_{min}} \frac{\ell}{\sqrt{2m\gamma + \ell^2}} \frac{du}{\sqrt{u_{min}^2 - u^2}} + \pi$$

Now, let $u = u_{min} \cos\psi$, u_{min} and 0 bound u

So $$\phi_m = \frac{\ell}{\sqrt{2m\gamma + \ell^2}} \int_{\frac{\pi}{2}}^{0} d\psi + \pi$$

$$= \pi - \frac{\pi}{2} \frac{\ell}{\sqrt{2m\gamma+\ell^2}}$$

Meaning

$$\theta = \left| \pi - 2\left(\pi - \frac{\pi}{2} \frac{\ell}{\sqrt{2m\gamma+\ell^2}}\right) \right|$$

$$= \left| -\pi + \frac{\ell\pi}{\sqrt{2m\gamma+\ell^2}} \right|$$

So as $\ell \to 0$, $\theta \to \pi$

and as $\ell \to \infty$, $\theta \to 0$ } logical

So write Θ as

$$\Theta = \pi - \frac{\pi \ell}{\sqrt{2m\gamma+\ell^2}}$$

We need $b(\theta)$ and $\ell = m v_0 b$

so $$\ell = \frac{\pi - \theta}{\pi}\left(2\pi\gamma + \ell^2\right)^{1/2}$$

$$\ell^2 = \left(\frac{\pi-\theta}{\pi}\right)^2\left(2\pi\gamma + \ell^2\right)$$

$$\ell^2\left(1 - \left(\frac{\pi-\theta}{\pi}\right)^2\right) = 2m\gamma\left(\frac{\pi-\theta}{\pi}\right)^2$$

$$\ell^2 = \frac{2m\gamma}{\left(\frac{\pi}{\pi-\theta}\right)^2 - 1}\,,$$ now use $\ell = m v_0 b^2$

$$b^2 = \frac{2m\gamma}{m^2 v_0^2}\,\frac{(\pi-\theta)^2}{\pi^2 - (\pi-\theta)^2}$$

$$= \frac{2m\gamma}{m^2 v_0^2}\,\frac{(\pi-\theta)^2}{\theta(2\pi-\theta)}$$

and
$$b(\theta) = \left[\frac{2m\gamma}{m^2 v_0^2}\left(\frac{(\pi-\theta)^2}{\theta(2\pi-\theta)}\right)\right]^{1/2}$$

need $\frac{db}{d\theta}$ to get $\frac{d\sigma}{d\Omega}$

$$\frac{db}{d\theta} = -\frac{\sqrt{2}\, M \pi^2 v_0^2 \left(\frac{\gamma(\pi-\theta)^2}{M v_0^2 (2\pi-\theta)\theta} \right)^{3/2}}{\gamma(\pi-\theta)^3}$$

Finally

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta} \left| \frac{db}{d\theta} \right|$$

$$= \frac{\pi^2 \gamma(\pi-\theta)}{\underbrace{\tfrac{1}{2} M v_0^2}_{=E} \sin\theta \, (2\pi-\theta)^2 \theta^2}$$

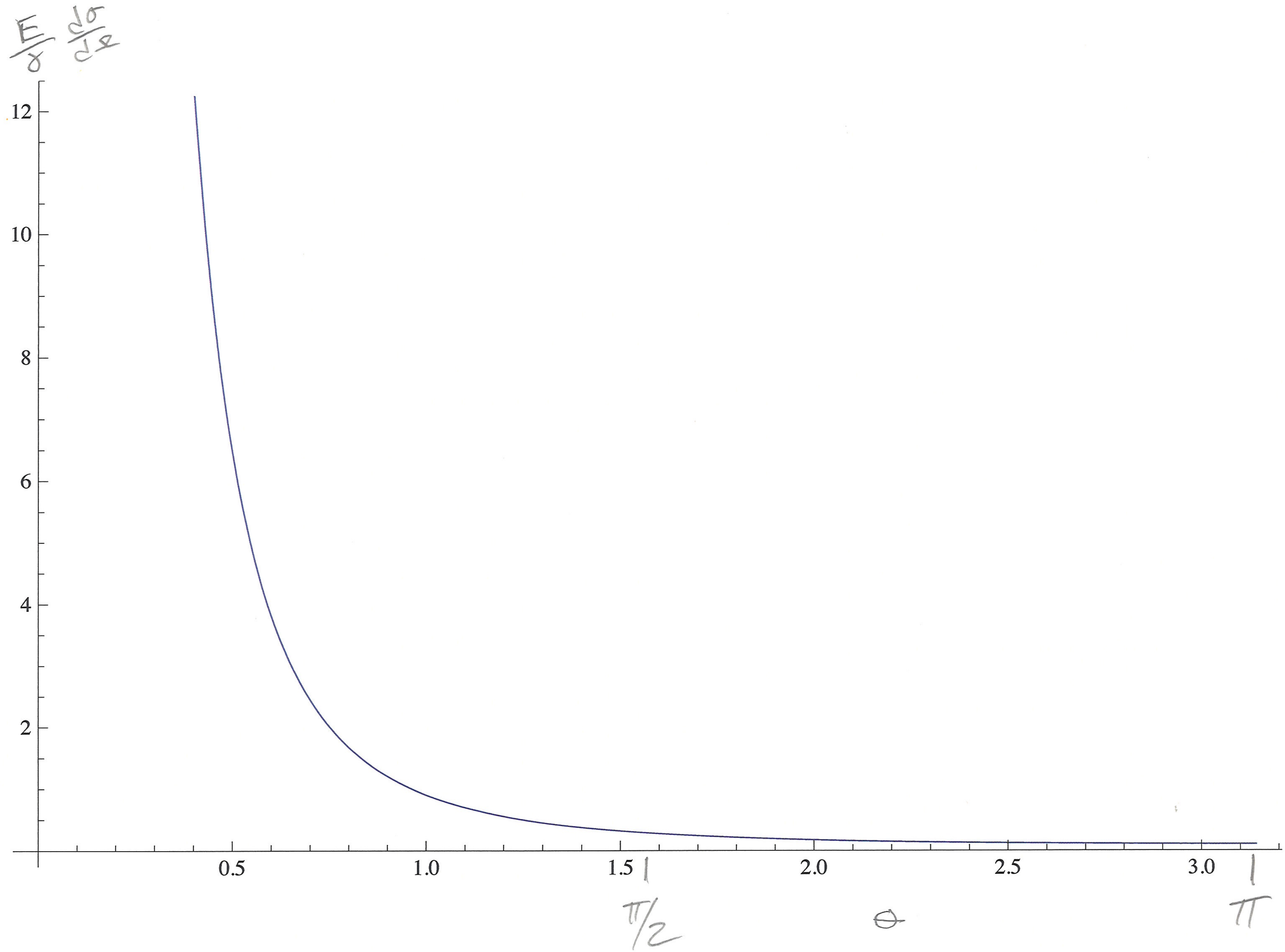
$$= \frac{\pi^2 \gamma(\pi-\theta)}{E \sin\theta \, (2\pi-\theta)^2 \theta^2}$$

See plot on next page.

Total Cross-Section is

$$\sigma_T = \int \frac{d\sigma}{d\Omega} d\Omega$$ here integrate over θ from 0 to 2π

Can see directly from plot that this will diverge to ∞ → All particles are deflected to some degree.



$\frac{E}{\gamma}\frac{d\sigma}{d\Omega}$
12
10
8
6
4
2
0.5
1.0
1.5
2.0
2.5
3.0
$\pi/2$
θ
π