

Derivatives and Integrals

In what follows, the letters u and v stand for any functions of x , and a and m are constants. To each of the indefinite integrals should be added an arbitrary constant of integration. The *Handbook of Chemistry and Physics* (CRC Press Inc.) gives a more extensive tabulation.

$$1. \frac{dx}{dx} = 1$$

$$2. \frac{d}{dx}(au) = a \frac{du}{dx}$$

$$3. \frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$$

$$4. \frac{d}{dx}x^m = mx^{m-1}$$

$$5. \frac{d}{dx} \ln x = \frac{1}{x}$$

$$6. \frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$7. \frac{d}{dx}e^x = e^x$$

$$8. \frac{d}{dx} \sin x = \cos x$$

$$9. \frac{d}{dx} \cos x = -\sin x$$

$$10. \frac{d}{dx} \tan x = \sec^2 x$$

$$11. \frac{d}{dx} \cot x = -\csc^2 x$$

$$12. \frac{d}{dx} \sec x = \tan x \sec x$$

$$13. \frac{d}{dx} \csc x = -\cot x \csc x$$

$$14. \frac{d}{dx} e^u = e^u \frac{du}{dx}$$

$$15. \frac{d}{dx} \sin u = \cos u \frac{du}{dx}$$

$$16. \frac{d}{dx} \cos u = -\sin u \frac{du}{dx}$$

$$1. \int dx = x$$

$$2. \int au \, dx = a \int u \, dx$$

$$3. \int (u + v) \, dx = \int u \, dx + \int v \, dx$$

$$4. \int x^m \, dx = \frac{x^{m+1}}{m+1} \quad (m \neq -1)$$

$$5. \int \frac{dx}{x} = \ln |x|$$

$$6. \int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx$$

$$7. \int e^x \, dx = e^x$$

$$8. \int \sin x \, dx = -\cos x$$

$$9. \int \cos x \, dx = \sin x$$

$$10. \int \tan x \, dx = \ln |\sec x|$$

$$11. \int \sin^2 x \, dx = \frac{1}{2}x - \frac{1}{4} \sin 2x$$

$$12. \int e^{-ax} \, dx = -\frac{1}{a} e^{-ax}$$

$$13. \int xe^{-ax} \, dx = -\frac{1}{a^2} (ax + 1)e^{-ax}$$

$$14. \int x^2 e^{-ax} \, dx = -\frac{1}{a^3} (a^2 x^2 + 2ax + 2)e^{-ax}$$

$$15. \int_0^\infty x^n e^{-ax} \, dx = \frac{n!}{a^{n+1}}$$

$$16. \int_0^\infty x^{2n} e^{-ax^2} \, dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1} a^n} \sqrt{\frac{\pi}{a}}$$

$$17. \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$$

$$18. \int \frac{x \, dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{(x^2 + a^2)^{1/2}}$$

$$19. \int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2(x^2 + a^2)^{1/2}}$$

$$20. \int_0^\infty x^{2n+1} e^{-ax^2} \, dx = \frac{n!}{2a^{n+1}} \quad (a > 0)$$

$$21. \int \frac{x \, dx}{x + d} = x - d \ln(x + d)$$

Binomial Theorem

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \dots \quad (x^2 < 1)$$

Exponential Expansion

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

Logarithmic Expansion

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \dots \quad (|x| < 1)$$

Trigonometric Expansions (θ in radians)

$$\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$$

$$\cos \theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots$$

$$\tan \theta = \theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \dots$$

Cramer's Rule

Two simultaneous equations in unknowns x and y ,

$$a_1x + b_1y = c_1 \quad \text{and} \quad a_2x + b_2y = c_2,$$

have the solutions

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{c_1b_2 - c_2b_1}{a_1b_2 - a_2b_1}$$

and

$$y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$$

Products of Vectors

Let \hat{i} , \hat{j} , and \hat{k} be unit vectors in the x , y , and z directions. Then

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1, \quad \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0,$$

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0,$$

$$\hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{k} = \hat{i}, \quad \hat{k} \times \hat{i} = \hat{j}.$$

Any vector \vec{a} with components a_x , a_y , and a_z along the x , y , and z axes can be written as

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}.$$

Let \vec{a} , \vec{b} , and \vec{c} be arbitrary vectors with magnitudes a , b , and c . Then

$$\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$$

$$(s\vec{a}) \times \vec{b} = \vec{a} \times (s\vec{b}) = s(\vec{a} \times \vec{b}) \quad (s = \text{a scalar}).$$

Let θ be the smaller of the two angles between \vec{a} and \vec{b} . Then

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} = a_xb_x + a_yb_y + a_zb_z = ab \cos \theta$$

$$\begin{aligned} \vec{a} \times \vec{b} &= -\vec{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \\ &= \hat{i} \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \hat{j} \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \hat{k} \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} \\ &= (a_yb_z - b_ya_z)\hat{i} + (a_zb_x - b_z a_x)\hat{j} \\ &\quad + (a_xb_y - b_x a_y)\hat{k} \end{aligned}$$

$$|\vec{a} \times \vec{b}| = ab \sin \theta$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b})$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$