Rotation HW Solutions
Phys303
Gladden

Chapter 10

4. If we make the units explicit, the function is

$$\theta = (4.0\text{ rad/s})t - (3.0\text{ rad/s}^2)t^2 + (1.0\text{ rad/s}^3)t^3$$

but generally we will proceed as shown in the problem—letting these units be understood. Also, in our manipulations we will generally not display the coefficients with their proper number of significant figures.

(a) Eq. 10-6 leads to

$$\omega = \frac{d}{dt}\left(4t - 3t^2 + t^3\right) = 4 - 6t + 3t^2.$$

Evaluating this at $t = 2$ s yields $\omega_2 = 4.0$ rad/s.

(b) Evaluating the expression in part (a) at $t = 4$ s gives $\omega_4 = 28$ rad/s.

(c) Consequently, Eq. 10-7 gives

$$\alpha_{\text{avg}} = \frac{\omega_4 - \omega_2}{4 - 2} = 12\text{ rad/s}^2.$$

(d) And Eq. 10-8 gives

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(4 - 6t + 3t^2\right) = -6 + 6t.$$

Evaluating this at $t = 2$ s produces $\alpha_2 = 6.0$ rad/s^2.

(e) Evaluating the expression in part (d) at $t = 4$ s yields $\alpha_4 = 18$ rad/s^2. We note that our answer for α_{avg} does turn out to be the arithmetic average of α_2 and α_4 but point out that this will not always be the case.

40. (a) Consider three of the disks (starting with the one at point O): ⊕OO . The first one (the one at point O – shown here with the plus sign inside) has rotational inertial (see item (c) in Table 10-2) $I = mR^2$. The next one (using the parallel-axis theorem) has

$$I = mR^2 + mh^2$$

where $h = 2R$. The third one has $I = mR^2 + m(4R)^2$. If we had considered five of the disks OO⊕OO with the one at O in the middle, then the total rotational inertia is

$$I = 5(mR^2) + 2(m(2R)^2 + m(4R)^2).$$

The pattern is now clear and we can write down the total I for the collection of fifteen disks:

$$I = 15(mR^2) + 2(m(2R)^2 + m(4R)^2 + m(6R)^2 + \ldots + m(14R)^2) = mR^2.$$

The generalization to N disks (where N is assumed to be an odd number) is

$$I = (2N^2 + 1)NmR^2.$$

In terms of the total mass ($m = M/15$) and the total length ($R = L/30$), we obtain

$$I = 0.083519ML^2 \approx (0.08352)(0.1000\text{ kg})(1.0000\text{ m})^2 = 8.352\times10^{-3}\text{ kg}\cdot\text{m}^2.$$

(b) Comparing to the formula (e) in Table 10-2 (which gives roughly $I = 0.08333\ ML^2$), we find our answer to part (a) is 0.22% lower.

44. (a) Using Table 10-2(c) and Eq. 10-34, the rotational kinetic energy is

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{4}(500\text{kg})(200\pi\ \text{ rad/s})^2(1.0\text{m})^2 = 4.9\times10^7\text{ J}.$$

(b) We solve $P = K/t$ (where P is the average power) for the operating time t.

$$t = \frac{K}{P} = \frac{4.9\times10^7\text{ J}}{8.0\times10^3\text{ W}} = 6.2\times10^3\text{ s}$$

which we rewrite as $t \approx 1.0\times10^2$ min.

67. From Table 10-2, the rotational inertia of the spherical shell is $2MR^2/3$, so the kinetic energy (after the object has descended distance h) is

$$K = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\omega^2_{\text{sphere}} + \frac{1}{2}I\omega^2_{\text{pulley}} + \frac{1}{2}mv^2.$$

Since it started from rest, then this energy must be equal (in the absence of friction) to the potential energy mgh with which the system started. We substitute v/r for the pulley's angular speed and v/R for that of the sphere and solve for v.

$$v = \sqrt{\frac{mgh}{\frac{1}{2}m + \frac{1}{2}\frac{I}{r^2} + \frac{M}{3}}} = \sqrt{\frac{2gh}{1 + (I/mr^2) + (2M/3m)}}$$

$$= \sqrt{\frac{2(9.8)(0.82)}{1 + 3.0\times 10^{-3}/((0.60)(0.050)^2) + 2(4.5)/3(0.60)}} = 1.4 \text{ m/s}$$

Chapter 11

15. (a) The derivation of the acceleration is found in §11-4; Eq. 11-13 gives

$$a_{\text{com}} = -\frac{g}{1 + I_{\text{com}}/MR_0^2}$$

where the positive direction is upward. We use $I_{\text{com}} = 950 \text{ g}\cdot\text{cm}^2$, M =120g, R_0 = 0.320 cm and g = 980 cm/s^2 and obtain

$$|a_{\text{com}}| = \frac{980}{1 + (950)/(120)(0.32)^2} = 12.5 \text{ cm/s}^2 \approx 13 \text{ cm/s}^2.$$

(b) Taking the coordinate origin at the initial position, Eq. 2-15 leads to $y_{\text{com}} = \frac{1}{2}a_{\text{com}}t^2$. Thus, we set y_{com} = – 120 cm, and find

$$t = \sqrt{\frac{2y_{\text{com}}}{a_{\text{com}}}} = \sqrt{\frac{2(-120\,\text{cm})}{-12.5 \text{ cm/s}^2}} = 4.38 \text{ s} \approx 4.4 \text{ s}.$$

(c) As it reaches the end of the string, its center of mass velocity is given by Eq. 2-11:

$$v_{com} = a_{com}t = \left(-12.5\ \text{cm/s}^2\right)\left(4.38\,\text{s}\right) = -54.8\ \text{cm/s}\,,$$

so its linear speed then is approximately 55 cm/s.

(d) The translational kinetic energy is

$$\tfrac{1}{2}mv_{com}^2 = \tfrac{1}{2}\left(0.120\ \text{kg}\right)\left(0.548\ \text{m/s}\right)^2 = 1.8\times 10^{-2}\ \text{J}\,.$$

(e) The angular velocity is given by $\omega = -\,v_{com}/R_0$ and the rotational kinetic energy is

$$\frac{1}{2}I_{com}\omega^2 = \frac{1}{2}I_{com}\frac{v_{com}^2}{R_0^2} = \frac{1}{2}\frac{\left(9.50\times 10^{-5}\ \text{kg}\cdot\text{m}^2\right)\left(0.548\ \text{m/s}\right)^2}{\left(3.2\times 10^{-3}\ \text{m}\right)^2}$$

which yields $K_{rot} = 1.4$ J.

(f) The angular speed is

$$\omega = \left|v_{com}\right|/R_0 = \left(0.548\,\text{m/s}\right)/\left(3.2\times 10^{-3}\ \text{m}\right) = 1.7\times 10^2\ \text{rad/s} = 27\,\text{rev/s}\,.$$

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29. (a) The acceleration vector is obtained by dividing the force vector by the (scalar) mass:

= /*m* = (3.00 m/s^2) – (4.00 m/s^2) + (2.00 m/s^2).

(b) Use of Eq. 11-18 leads directly to

= (42.0 kg·m^2/s) + (24.0 kg·m^2/s) + (60.0 kg·m^2/s).

(c) Similarly, the torque is

$\vec{\tau} = \vec{r}\times\vec{F}$ = (–8.00 N·m) – (26.0 N·m) – (40.0 N·m).

(d) We note (using the Pythagorean theorem) that the magnitude of the velocity vector is 7.35 m/s and that of the force is 10.8 N. The dot product of these two vectors is · = – 48 (in SI units). Thus, Eq. 3-20 yields

$$\theta = \cos^{-1}[-48.0/(7.35\times 10.8)] = 127°.$$

33. (a) We note that

$$\vec{v} = \frac{d\vec{r}}{dt} = 8.0t \ - (2.0 + 12t)$$

with SI units understood. From Eq. 11-18 (for the angular momentum) and Eq. 3-30, we find the particle's angular momentum is $8t^2$. Using Eq. 11-23 (relating its time-derivative to the (single) torque) then yields $= 48t$.

(b) From our (intermediate) result in part (a), we see the angular momentum increases in proportion to t^2.

64. (a) We choose clockwise as the negative rotational sense and rightwards as the positive translational direction. Thus, since this is the moment when it begins to roll smoothly, Eq. 11-2 becomes $v_{\text{com}} = -R\omega = (-0.11 \text{ m})\omega$.

This velocity is positive-valued (rightward) since ω is negative-valued (clockwise) as shown in Fig. 11-57.

(b) The force of friction exerted on the ball of mass m is $-\mu_k mg$ (negative since it points left), and setting this equal to ma_{com} leads to

$$a_{\text{com}} = -\mu g = -(0.21)\left(9.8 \text{ m/s}^2\right) = -2.1 \text{ m/s}^2$$

where the minus sign indicates that the center of mass acceleration points left, opposite to its velocity, so that the ball is decelerating.

(c) Measured about the center of mass, the torque exerted on the ball due to the frictional force is given by $\tau = -\mu mgR$. Using Table 10-2(f) for the rotational inertia, the angular acceleration becomes (using Eq. 10-45)

$$\alpha = \frac{\tau}{I} = \frac{-\mu mgR}{2mR^2/5} = \frac{-5\mu g}{2R} = \frac{-5(0.21)(9.8)}{2(0.11)} = -47 \text{ rad/s}^2$$

where the minus sign indicates that the angular acceleration is clockwise, the same direction as ω (so its angular motion is "speeding up'').

(d) The center-of-mass of the sliding ball decelerates from $v_{\text{com},0}$ to v_{com} during time t according to Eq. 2-11: $v_{\text{com}} = v_{\text{com},0} - \mu gt$. During this time, the angular speed of the ball increases (in magnitude) from zero to $|\omega|$ according to Eq. 10-12:

$$|\omega| = |\alpha|t = \frac{5\mu gt}{2R} = \frac{v_{\text{com}}}{R}$$

where we have made use of our part (a) result in the last equality. We have two equations involving v_{com}, so we eliminate that variable and find

$$t = \frac{2v_{\text{com},0}}{7\mu g} = \frac{2(8.5)}{7(0.21)(9.8)} = 1.2 \text{ s.}$$

(e) The skid length of the ball is (using Eq. 2-15)

$$\Delta x = v_{\text{com},0}t - \frac{1}{2}(\mu g)t^2 = (8.5)(1.2) - \frac{1}{2}(0.21)(9.8)(1.2)^2 = 8.6 \text{ m.}$$

(f) The center of mass velocity at the time found in part (d) is

$$v_{\text{com}} = v_{\text{com},0} - \mu gt = 8.5 - (0.21)(9.8)(1.2) = 6.1 \text{ m/s.}$$