Physics 303
Solution for HW03
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Chapter 7

10. The change in kinetic energy can be written as

$$\Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}m(2a\Delta x) = ma\Delta x$$

where we have used $v_f^2 = v_i^2 + 2a\Delta x$ from Table 2-1. From Fig. 7-27, we see that $\Delta K = (0-30)\ \text{J} = -30\ \text{J}$ when $\Delta x = +5\ \text{m}$. The acceleration can then be obtained as

$$a = \frac{\Delta K}{m\Delta x} = \frac{(-30\ \text{J})}{(8.0\ \text{kg})(5.0\ \text{m})} = -0.75\ \text{m/s}^2.$$

The negative sign indicates that the mass is decelerating. From the figure, we also see that when $x = 5$ m the kinetic energy becomes zero, implying that the mass comes to rest momentarily. Thus,

$$v_0^2 = v^2 - 2a\Delta x = 0 - 2(-0.75\ \text{m/s}^2)(5.0\ \text{m}) = 7.5\ \text{m}^2/\text{s}^2,$$

or $v_0 = 2.7\ \text{m/s}$. The speed of the object when $x = -3.0$ m is

$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{7.5 + 2(-0.75)(-3.0)} = \sqrt{12} = 3.5\ \text{m/s}.$$

11. We choose +*x* as the direction of motion (so $\vec{a}$ and $\vec{F}$ are negative-valued).

(a) Newton's second law readily yields $\vec{F} = (85\,\text{kg})(-2.0\,\text{m/s}^2)$ so that

$$F = |\vec{F}| = 1.7\times10^2\ \text{N}.$$

(b) From Eq. 2-16 (with $v = 0$) we have

$$0 = v_0^2 + 2a\Delta x \quad\Rightarrow\quad \Delta x = -\frac{(37\,\text{m/s})^2}{2(-2.0\,\text{m/s}^2)} = 3.4\times10^2\ \text{m}.$$

Alternatively, this can be worked using the work-energy theorem.

(c) Since $\vec{F}$ is opposite to the direction of motion (so the angle ϕ between $\vec{F}$ and $\vec{d} = \Delta x$ is 180°) then Eq. 7-7 gives the work done as $W = -F\Delta x = -5.8\times10^4$ J.

(d) In this case, Newton's second law yields $\vec{F} = (85\,\text{kg})(-4.0\,\text{m/s}^2)$ so that $F = |\vec{F}| = 3.4\times10^2$ N.

(e) From Eq. 2-16, we now have

$$\Delta x = -\frac{(37\,\text{m/s})^2}{2(-4.0\,\text{m/s}^2)} = 1.7\times10^2\ \text{m}.$$

(f) The force $\vec{F}$ is again opposite to the direction of motion (so the angle ϕ is again 180°) so that Eq. 7-7 leads to $W = -F\Delta x = -5.8\times10^4$ J. The fact that this agrees with the result of part (c) provides insight into the concept of work.

15. Using the work-kinetic energy theorem, we have

$$\Delta K = W = \vec{F}\cdot\vec{d} = Fd\cos\phi$$

In addition, $F = 12$ N and $d = \sqrt{(2.00)^2 + (-4.00)^2 + (3.00)^2} = 5.39$ m.

(a) If $\Delta K = +30.0$ J, then

$$\phi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left(\frac{30.0}{(12.0)(5.39)}\right) = 62.3°\,.$$

(b) $\Delta K = -30.0$ J, then

$$\phi = \cos^{-1}\left(\frac{\Delta K}{Fd}\right) = \cos^{-1}\left(\frac{-30.0}{(12.0)(5.39)}\right) = 118°$$

21. Eq. 7-15 applies, but the wording of the problem suggests that it is only necessary to examine the contribution from the rope (which would be the "W_a" term in Eq. 7-15):

$$W_a = -(50\text{ N})(0.50\text{ m}) = \;\; -25\text{ J}$$

(the minus sign arises from the fact that the pull from the rope is anti-parallel to the direction of motion of the block). Thus, the kinetic energy would have been 25 J greater if the rope had not been attached (given the same displacement).

35. We choose to work this using Eq. 7-10 (the work-kinetic energy theorem). To find the initial and final kinetic energies, we need the speeds, so

$$v = \frac{dx}{dt} = 3.0 - 8.0t + 3.0t^2$$

in SI units. Thus, the initial speed is v_i = 3.0 m/s and the speed at t = 4 s is v_f = 19 m/s. The change in kinetic energy for the object of mass m = 3.0 kg is therefore

$$\Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = 528\text{ J}$$

which we round off to two figures and (using the work-kinetic energy theorem) conclude that the work done is $W = 5.3\times10^2$ J.

37. (a) We first multiply the vertical axis by the mass, so that it becomes a graph of the applied force. Now, adding the triangular and rectangular "areas" in the graph (for $0 \le x \le 4$) gives 42 J for the work done.

(b) Counting the "areas" under the axis as negative contributions, we find (for $0 \le x \le 7$) the work to be 30 J at x = 7.0 m.

(c) And at x = 9.0 m, the work is 12 J.

(d) Eq. 7-10 (along with Eq. 7-1) leads to speed v = 6.5 m/s at x = 4.0 m. Returning to the original graph (where a was plotted) we note that (since it started from rest) it has received acceleration(s) (up to this point) only in the $+x$ direction and consequently must have a velocity vector pointing in the $+x$ direction at x = 4.0 m.

(e) Now, using the result of part (b) and Eq. 7-10 (along with Eq. 7-1) we find the speed is 5.5 m/s at x = 7.0 m. Although it has experienced some deceleration during the $0 \le x \le 7$ interval, its velocity vector still points in the $+x$ direction.

(f) Finally, using the result of part (c) and Eq. 7-10 (along with Eq. 7-1) we find its speed v = 3.5 m/s at x = 9.0 m. It certainly has experienced a significant amount of deceleration during the $0 \le x \le 9$ interval; nonetheless, its velocity vector *still* points in the $+x$ direction.

41. The power associated with force $\vec{F}$ is given by $P = \vec{F} \cdot \vec{v}$, where $\vec{v}$ is the velocity of the object on which the force acts. Thus,

$$P = \vec{F} \cdot \vec{v} = Fv\cos\phi = (122\ \text{N})(5.0\ \text{m/s})\cos 37° = 4.9\times10^2\ \text{W}.$$

48. (a) With SI units understood, the object's displacement is

$$\vec{d} = \vec{d}_f - \vec{d}_i = -8.00\,\hat{\text{i}} + 6.00\,\hat{\text{j}} + 2.00\,\hat{\text{k}}\ .$$

Thus, Eq. 7-8 gives $W = \vec{F}\cdot\vec{d} = (3.00)(-8.00) + (7.00)(6.00) + (7.00)(2.00) = 32.0$ J.

(b) The average power is given by Eq. 7-42:

$$P_{\text{avg}} = \frac{W}{t} = \frac{32.0}{4.00} = 8.00\ \text{W}.$$

(c) The distance from the coordinate origin to the initial position is $d_i = \sqrt{(3.00)^2 + (-2.00)^2 + (5.00)^2} = 6.16\ \text{m}$, and the magnitude of the distance from the coordinate origin to the final position is $d_f = \sqrt{(-5.00)^2 + (4.00)^2 + (7.00)^2} = 9.49\ \text{m}$. Their scalar (dot) product is

$$\vec{d}_i \cdot \vec{d}_f = (3.00)(-5.00) + (-2.00)(4.00) + (5.00)(7.00) = 12.0\ \text{m}^2.$$

Thus, the angle between the two vectors is

$$\phi = \cos^{-1}\left(\frac{\vec{d}_i\cdot\vec{d}_f}{d_i d_f}\right) = \cos^{-1}\left(\frac{12.0}{(6.16)(9.49)}\right) = 78.2°.$$

49. From Eq. 7-32, we see that the "area" in the graph is equivalent to the work done. We find the area in terms of rectangular [length × width] and triangular [$\frac{1}{2}$ base × height] areas and use the work-kinetic energy theorem appropriately. The initial point is taken to be $x = 0$, where $v_0 = 4.0$ m/s.

(a) With $K_i = \frac{1}{2}mv_0^2 = 16$ J, we have

$$K_3 - K_0 = W_{0<x<1} + W_{1<x<2} + W_{2<x<3} = -4.0 \text{ J}$$

so that K_3 (the kinetic energy when x = 3.0 m) is found to equal 12 J.

(b) With SI units understood, we write $W_{3<x<x_f}$ as $F_x \Delta x = (-4.0)(x_f - 3.0)$ and apply the work-kinetic energy theorem:

so that the requirement $Kx_f = 8.0$ J leads to $x_f = 4.0$ m.

(c) As long as the work is positive, the kinetic energy grows. The graph shows this situation to hold until x = 1.0 m. At that location, the kinetic energy is

$$K_1 = K_0 + W_{0<x<1} = 16 \text{ J} + 2.0 \text{ J} = 18 \text{ J}.$$