# Physics 214 HW04 Solutions
# Gladden, Spring 2008

## CHAPTER 19: DC Circuits

## Answers to Questions

1. The birds are safe because they are not grounded. Both of their legs are essentially at the same voltage (the only difference being due to the small resistance of the wire between their feet), and so there is no current flow through their bodies since the potential difference across their legs is very small. If you lean a metal ladder against the power line, you are making essentially a short circuit from the high potential wire to the low potential ground. A large current will flow at least momentarily, and that large current will be very dangerous to anybody touching the ladder.

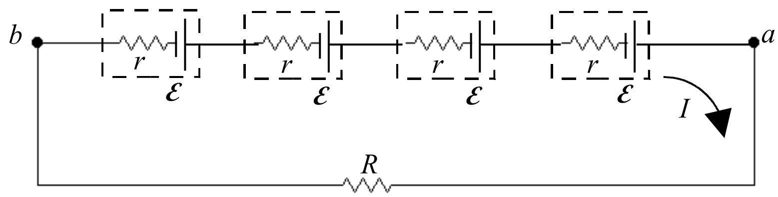
4. If the lightbulbs are in series, each will have the same current. The power dissipated by the bulb as heat and light is given by $P = I^2 R$. Thus the bulb with the higher resistance $(R_2)$ will be brighter. If the bulbs are in parallel, each will have the same voltage. The power dissipated by the bulb as heat and light is given by $P = V^2/R$. Thus the bulb with the lower resistance $(R_1)$ will be brighter.

6. The power output from a resistor is given by $P = V^2/R$. To maximize this value, the voltage needs to be as large as possible and the resistance as small as possible. That can be accomplished by putting the two batteries in series, and then connecting the two resistors in parallel to each other, across the full 2-battery voltage.

15. The energy stored in a capacitor network can be calculated by $PE = \frac{1}{2}CV^2$. Since the voltage for the capacitor network is the same in this problem for both configurations, the configuration with the highest equivalent capacitance will store the most energy. The parallel combination has the highest equivalent capacitance, and so stores the most energy. Another way to consider this is that the total stored energy is the sum of the quantity $PE = \frac{1}{2}CV^2$ for each capacitor. Each capacitor has the same capacitance, but in the parallel circuit, each capacitor has a larger voltage than in the series circuit. Thus the parallel circuit stores more energy.

## Solutions to Problems

2. See the circuit diagram below. The current in the circuit is $I$. The voltage $V_{ab}$ is given by Ohm's law to be $V_{ab} = IR$. That same voltage is the terminal voltage of the series EMF.




$$V_{ab} = (\mathcal{E} - Ir) + (\mathcal{E} - Ir) + (\mathcal{E} - Ir) + (\mathcal{E} - Ir) = 4(\mathcal{E} - Ir) \quad \text{and} \quad V_{ab} = IR$$

$$4(\mathcal{E} - Ir) = IR \quad \rightarrow \quad r = \frac{\mathcal{E} - \frac{1}{4}IR}{I} = \frac{(1.5\,\text{V}) - \frac{1}{4}(0.45\,\text{A})(12\,\Omega)}{0.45\,\text{A}} = \boxed{0.33\,\Omega}$$

9. (*a*) The maximum resistance is made by combining the resistors in series.

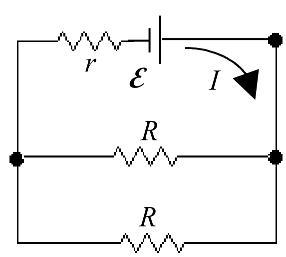
$$R_{eq} = R_1 + R_2 + R_3 = 680\,\Omega + 940\,\Omega + 1200\,\Omega = \boxed{2820\,\Omega}$$

(*b*) The minimum resistance is made by combining the resistors in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \rightarrow$$

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{680\,\Omega} + \frac{1}{940\,\Omega} + \frac{1}{1200\,\Omega}\right)^{-1} = \boxed{3.0 \times 10^2\,\Omega}$$

12. The resistance of each bulb can be found from its power rating.




$$P = \frac{V^2}{R} \quad \rightarrow \quad R = \frac{V^2}{P} = \frac{(12.0\,\text{V})^2}{3.0\,\text{W}} = 48\,\Omega$$

Find the equivalent resistance of the two bulbs in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R} \quad \rightarrow \quad R_{eq} = \frac{R}{2} = \frac{48\,\Omega}{2} = 24\,\Omega$$

The terminal voltage is the voltage across this equivalent resistance.
Use that to find the current drawn from the battery.

$$V_{ab} = IR_{eq} \quad \rightarrow \quad I = \frac{V_{ab}}{R_{eq}} = \frac{V_{ab}}{R/2} = \frac{2V_{ab}}{R}$$

Finally, use the terminal voltage and the current to find the internal resistance, as in Eq. 19-1.

$$V_{ab} = \mathcal{E} - Ir \quad \rightarrow \quad r = \frac{\mathcal{E} - V_{ab}}{I} = \frac{\mathcal{E} - V_{ab}}{\left(\frac{2V_{ab}}{R}\right)} = R\frac{\mathcal{E} - V_{ab}}{2V_{ab}} = (48\,\Omega)\frac{12.0\,\text{V} - 11.8\,\text{V}}{2(11.8\,\text{V})} = \boxed{0.4\,\Omega}$$

15. Each bulb will get one-eighth of the total voltage, and so $V_{\text{bulb}} = \dfrac{V_{\text{tot}}}{8}$. Use that voltage and the power dissipated by each bulb to calculate the resistance of a bulb.

$$P_{\text{bulb}} = \frac{V_{\text{bulb}}^2}{R} \rightarrow R = \frac{V_{\text{bulb}}^2}{P} = \frac{V_{\text{tot}}^2}{64P} = \frac{(110\,\text{V})^2}{64(7.0\,\text{W})} = \boxed{27\,\Omega}$$

24. Apply Kirchhoff's loop rule to the circuit starting at the upper left corner of the circuit diagram, in order to calculate the current. Assume that the current is flowing clockwise.

$$-I(1.0\,\Omega) + 18\,\text{V} - I(6.6\,\Omega) - 12\,\text{V} - I(2.0\,\Omega) = 0 \rightarrow I = \frac{6\,\text{V}}{9.6\,\Omega} = 0.625\text{A}$$

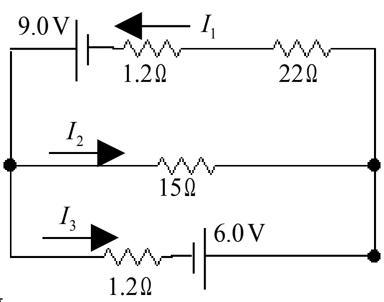
The terminal voltage for each battery is found by summing the potential differences across the internal resistance and EMF from left to right. Note that for the 12 V battery, there is a voltage gain going across the internal resistance from left to right.

$$18\,\text{V battery: } V_{\text{terminal}} = -I(1.0\,\Omega) + 18\,\text{V} = -(0.625\,\text{A})(1.0\,\Omega) + 18\,\text{V} = \boxed{17.4\,\text{V}}$$

$$12\,\text{V battery: } V_{\text{terminal}} = I(2.0\,\Omega) + 12\,\text{V} = (0.625\,\text{A})(2.0\,\Omega) + 12\,\text{V} = \boxed{13.3\,\text{V}}$$

28. There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches on the left of the circuit.

$$I_1 = I_2 + I_3$$




Another equation comes from Kirchhoff's loop rule applied to the outer loop, starting at the lower left corner, and progressing counterclockwise.

$$-I_3(1.2\,\Omega) + 6.0\,\text{V} - I_1(22\,\Omega) - I_1(1.2\,\Omega) + 9.0\,\text{V} = 0 \rightarrow$$

$$15 = 23.2I_1 + 1.2I_3$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the lower left corner, and progressing counterclockwise.

$$-I_3(1.2\,\Omega) + 6.0\,\text{V} + I_2(15\,\Omega) = 0 \rightarrow 6 = -15I_2 + 1.2I_3$$

Substitute $I_1 = I_2 + I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$15 = 23.2I_1 + 1.2I_3 = 23.2(I_2 + I_3) + 1.2I_3 = 23.2I_2 + 24.4I_3 \;;\; 6 = -15I_2 + 1.2I_3$$

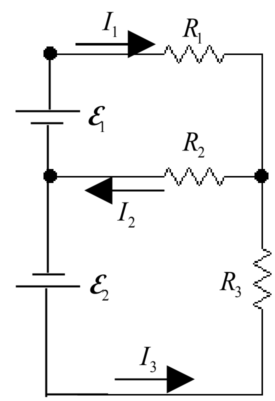
Solve the bottom loop equation for $I_2$ and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$6 = -15I_2 + 1.2I_3 \;\rightarrow\; I_2 = \frac{-6 + 1.2I_3}{15}$$

$$15 = 23.2I_2 + 24.4I_3 = 23.2\left(\frac{-6+1.2I_3}{15}\right) + 24.4I_3 \;\rightarrow\; 225 = -138 + 27.84I_3 + 366I_3 \;\rightarrow$$

$$I_3 = \frac{363}{393.84} = 0.9217\text{ A} \;;\; I_2 = \frac{-6+1.2I_3}{15} = \frac{-6+1.2(0.9217)}{15} = -0.3263\text{ A} \approx \boxed{0.33\text{ A , left}}$$

$$I_1 = I_2 + I_3 = 0.5954\text{ A} \approx \boxed{0.60\text{ A , left}}$$




36. The maximum capacitance is found by connecting the capacitors in parallel.

$$C_{\max} = C_1 + C_2 + C_3 = 3.2\times10^{-9}\text{F} + 7.5\times10^{-9}\text{F} + 1.00\times10^{-8}\text{F} = \boxed{2.07\times10^{-8}\text{F in parallel}}$$

The minimum capacitance is found by connecting the capacitors in series.

$$C_{\min} = \left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1} = \left(\frac{1}{3.2\times10^{-9}\text{F}}+\frac{1}{7.5\times10^{-9}\text{F}}+\frac{1}{1.00\times10^{-8}\text{F}}\right)^{-1} = \boxed{1.83\times10^{-9}\text{F in series}}$$

40. Capacitors in parallel add linearly, and so adding a capacitor in parallel will increase the net capacitance without removing the $5.0\mu\text{F}$ capacitor.

$$5.0\mu\text{F} + C = 16\mu\text{F} \;\rightarrow\; C = \boxed{11.0\mu\text{F connected in parallel}}$$

50. (*a*) From Eq. 19-7. the product *RC* is equal to the time constant.

$$\tau = RC \;\rightarrow\; C = \frac{\tau}{R} = \frac{35.0\times10^{-6}\text{s}}{15.0\times10^{3}\Omega} = \boxed{2.33\times10^{-9}\text{F}}$$

(*b*) Since the battery has an EMF of 24.0 V, if the voltage across the resistor is 16.0 V, the voltage across the capacitor will be 8.0 V as it charges. Use the expression for the voltage across a charging capacitor.

$$V_C = \mathcal{E}\left(1-e^{-t/\tau}\right) \;\rightarrow\; e^{-t/\tau} = \left(1-\frac{V_C}{\mathcal{E}}\right) \;\rightarrow\; -\frac{t}{\tau} = \ln\left(1-\frac{V_C}{\mathcal{E}}\right) \;\rightarrow$$

$$t = -\tau\ln\left(1-\frac{V_C}{\mathcal{E}}\right) = -\left(35.0\times10^{-6}\text{s}\right)\ln\left(1-\frac{8.0\text{ V}}{24.0\text{ V}}\right) = \boxed{1.42\times10^{-5}\text{s}}$$