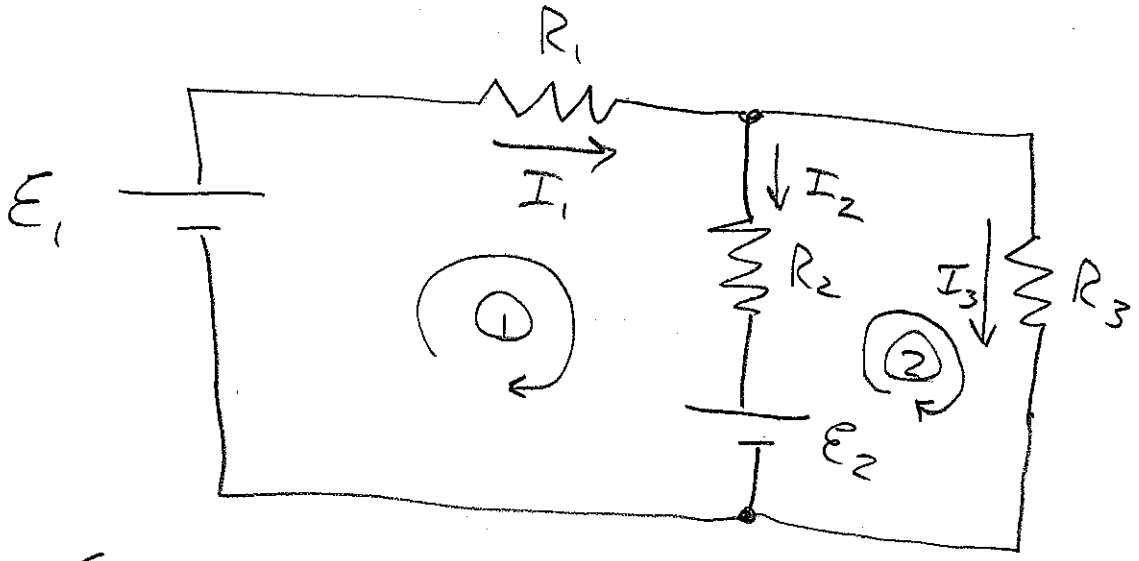
Hard Circuit Problem

from review session



assume $R_1 \neq R_2 \neq R_3$

Say you are given $R_1, R_2, R_3, \mathcal{E}_1 + \mathcal{E}_2$ and need to find I_1, I_2, I_3

3 equations

Loop ①: $\mathcal{E}_1 - I_1 R_1 - I_2 R_2 - \mathcal{E}_2 = 0$

Loop ②: $\mathcal{E}_2 + I_2 R_2 - I_3 R_3 = 0$

Junction: $I_1 = I_2 + I_3$

Now Solve!

From ②: $I_2 R_2 = I_3 R_3 - \mathcal{E}_2$, plug into ①

to get $\mathcal{E}_1 - I_1 R_1 - I_3 R_3 + \cancel{\mathcal{E}_2} - \cancel{\mathcal{E}_2} = 0$

Now solve this for I_1

$I_1 = \dfrac{\mathcal{E}_1 - I_3 R_3}{R_1}$ $= I_2 + I_3$ ← by Junction

Now Solve for I_2:

$$I_2 = \frac{\mathcal{E}_1}{R_1} - \frac{R_3}{R_1} I_3 - I_3$$

so $$I_2 R_2 = \frac{R_2}{R_1}\mathcal{E}_1 - \frac{R_2 R_3}{R_1} I_3 - R_2 I_3$$

and plug this into Loop ② eqn and Solve for I_3

$$\mathcal{E}_2 + \frac{R_2}{R_1}\mathcal{E}_1 - \frac{R_2 R_3}{R_1} I_3 - R_2 I_3 - I_3 R_3 = 0$$

Collect I_3 terms and Solve:

$$\mathcal{E}_2 + \frac{R_2}{R_1}\mathcal{E}_1 = \left[\frac{R_2 R_3}{R_1} + R_2 + R_3\right] I_3$$

so $$I_3 = \frac{\mathcal{E}_2 + \frac{R_2}{R_1}\mathcal{E}_1}{\left[\frac{R_2 R_3}{R_1} + R_2 + R_3\right]}$$

you know numbers for all this, so Calculate it.

Then plug I_3 value into loop ② eqn, and Solve for I_2 and finally I_1 by $I_1 = I_2 + I_3$

... Easy as pie!