The Care and Feeding of a Physics Problem Set

Itai Seggev

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1 Introduction

Dear Student:

The document you hold in your hands is quite important. It contains detailed instructions on how to raise your problem set so that it grows up to be strong, healthy, and high scoring. As an added benefit, the suggestions in this document may even aid the learning process. In the sections that follow I will describe various components which make for a good solution. These rules are not arbitrary nor meant solely for my benefit. They are things which I started doing over the course of my college career, sometimes voluntarily and other times involuntarily, and which helped me master the material. I will therefore explain my rationale for including them. I hope you will approach them with an open mind and try to see how they are helpful.

The above table of contents doubles as a summary of the components of a good problem set. As the name suggests, the required components in Section 2 are expected, and you will lose points if you fail to incorporate those elements into your homework. The recommended components in Section 3 will, in my judgment, greatly benefit you , but I will not mark off for their absence. In order to clarify the rules, Section 4 gives a sample problem with different solutions. It then explains how each of these solutions might be graded. Finally, some last thoughts appear in Section 5.

2 Required Components

2.1 Explain your work clearly, in English

English is the water for your problem set: it is absolutely essential for survival. Like physical water, it is possible to drink too much, but no one ever does.

This is the single most important part of a good solution. You should assume that I do not know how to solve the problem and that you need to explain it to me. If your solution differs significantly from mine, this assumption may actually be correct. You will not receive full credit if I do not completely understand how you arrived at your answer. This is because getting the correct answer does not indicate that your method of solution is correct, as shown by the following classic calculation:

$$\frac{16}{64} = \frac{1\cancel{6}}{\cancel{6}4} = \frac{1}{4}.$$

It is an unfortunate fact that there are many wrong ways to get the right answer. Explaining your work so that I understand your logic is the only way to convince me that you got the right answer by valid means.

However, explaining your work clearly has benefits for you completely unrelated to your grade. As you write up your solution you may find that you do not fully understand certain steps. By writing as precise a solution as possible, you will discover exactly what parts of the material still confuse you, and will thus be in a better position to seek help (from the book, a friend, or myself). You may also find that steps which seem obvious to you when you were solving the problem are no longer obvious when you are looking at your work later, for example when you are studying for the test. By writing everything down you will help yourself understand what you are doing.

2.2 Have a high E/M (English/Math) ratio

Just as you should drink at least 2 quarts of water each day, your problem set needs an E/M ratio of at least 2 for optimal health.

This is really the same as 2.1, but it is sufficiently important that it is worth saying a second time. A substantial portion of your solution should be in English. A sequence of equations by itself transmits little information. Short helper phrases like "therefore," "on the other hand," "by definition," and "substituting in from... yields" can work wonders in clarifying the logical structure of a solution. My solutions almost never have an E/M ratio less than one, and it typically hovers between 2 and 3 (in terms of vertical space consumed by each on the page). Your solutions should be comparable.

2.3 Indicate your assumptions explicitly

Problem sets should always be vaccinated against unclear wording, hidden assumptions, and rabies.

It is a sad truth that physics problems are not always as precisely stated as they should be. Authors may make hidden assumptions which are necessary to make the problem tractable. Other times, the may ask for estimates. In this case you have to make certain guesses or assumptions along the way in order to solve them problem (and there will be multiple correct answers, since there is more than one "reasonable" estimate). By stating your assumptions up front, you help me understand your solution, especially if you interpreted the problem differently than I did. More importantly, you may be cluing yourself in to the fact that you can use simplification techniques such as the binomial expansion.

2.4 Define your variables

Always start off your problem set's day with a balanced breakfast of variable definitions.

Define your variables unless they are defined in the problem statement. Almost every variable has multiple "conventional" uses. Even if it does not, different people will usually manage to find different conventions for a given variable, such as different origins, different orientations of the axes, or even different parts of the system. You may even find that you no longer remember which conventions you used when you reread your solution later. Therefore, when in doubt, define, as this increases the odds that I will understand your solution correctly. Including appropriate illustrations of your variable definitions is extremely helpful in this regard.

2.5 Include proper diagrams whenever possible

Supplement your problem set's diet with diagrammatic multivitamins when necessary. Small but powerful, a picture can be worth a thousand milligrams of words.

A proper diagram (i.e., one that is legible, has labeled axes, and is drawn with sufficient care to convey the essential quantities in the problem) can immediately clarify your variable definitions for me and provide insight into the problem for you. In mechanics, we use a special kind of diagram, called a *free-body diagram*, to show how the different forces in a problem act. Once we learn about these, you will be expected to include them on every problem where you need to find a force or torque.

2.6 Solve the problem symbolically

Even though your problem set may want to run ahead to the numbers in the park, it needs to learn to stay with you as you walk along the symbolic trail.

While there are occasional exceptions, numbers have no business appearing until the last line of a solution. Even in exceptional cases, a variable should be assigned to the intermediate answer rather than simply using the number. After all, if the numbers are plugged in, there is no way to take limits as described in 3.3, below. Non-symbolic computations also make it more difficult to follow the logic and reduce the opportunity for partial credit. If you just have a horribly wrong number without any symbolic expressions, I may have no idea whether you simply punched the numbers incorrectly into your calculator or whether your entire method of solution is incorrect.

2.7 Use a reasonable number of digits

Digits form the fat of a solution's diet: some is absolutely essential, but too much will quickly lead to poor health.

The final answer should have the number of significant digits dictated by the quantities in the problem statement. This is rarely more than three, no matter how many digits the calculator gives. Never write a number with twelve digits. The answer is not 0.99999999999978; rather, it is 1.00 (or possibly 2.2×10^{-13} if the requested result is the deviation from unity). This both expresses the correct number of significant digits and is much easier to read. Besides clarity, keeping track of significant digits will often indicate at what point a particular expansion, such as the binomial expansion, can be truncated. By this I mean that if the answer has three significant digits, and the second order correction is of order 10^{-6}, you can be certain that you may safely truncate the series at first order.

2.8 Apply the "Gravity Does Not Point Up" test

Pay attention to the behaviour of your problem set. If it becomes strange or unexpected, it may be ill and require medical attention.

A professor once told me, "gravity does not point up. If you finish a problem and discover that gravity is pointing up, you've probably done something wrong." The wonderful thing about physics is that it deals with the world around us, and thus we often have clues as to what the answer should be. After you have gotten your answer, pause and consider its reasonableness. If you compute that the time required to bring a car to a full stop is $1,000,000$ times the age of the universe, you have likely made an error. Go back and look for that error. If you cannot find it, indicate that your answer is unreasonable and why. Such a statement indicates a greater level of understanding of the material, and hence receives more partial credit, than blindly writing down a clearly wrong answer.

2.9 Get your units right

If your problem set exhibits the aggression of conflicting units, seek the help of your veterinarian immediately!

The first step of the "Gravity" test is to make sure the units of your answer are right. If the problem asks for a velocity, the units cm/s, m/s, and furlongs/fortnight are all possibly correct, but kg-m/s is not. Answers which have the wrong dimensions are necessarily wrong—go back and check what happened. Further, numbers should always have their units attached to them (unless they are dimensionless). This makes it much easier to find out where the units went bad. Notice that you do not need to plug in numbers to see if the units in your answer are right. Each variable has a dimension associated to it. Dimensional analysis of your symbolic answer should tell you if it has the right units. If it does not, check the dimensional consistency of each previous equation.

3 Recommended Components

3.1 Copy the problem statement

Problem sets are happiest when all their parts can be found in one place

Put a copy of the problem on the same page as your solution. This can be either a literal copy which is pasted (or transcribed) onto the paper or a paraphrasing of the problem statement. The advantage of the former is that it is quicker and ensures that the copy is correct. The advantage of the latter is that the process of paraphrasing may help you understand the problem better. In either event, attaching a copy makes studying for exams easier, as both problem and solution can be found in one place.

3.2 Use only one side of the paper

Problem sets enjoy pet doors which allow them to move freely from the house to the back yard.

Using one side of the paper makes it easier to follow a solution if it stretches across more than one page. However, if you worry about killing trees, do not hesitate to use both sides of the paper.

3.3 Examine various limits of your answer

Kant's categorical imperative applies just as well to pet care as it does to moral philosophy: things which are good for your pet in one situation should still be good for it in many other situations.

One of the best ways to test the reasonableness of your answer is to take various limits of the final answer. It may not always be possible to look at the answer and determine whether it is sensible. However, there will usually be special cases in which the answer should be easy to understand. Figure out what happens to your answer when one of the masses goes to zero, when the two masses become equal, or when the incline plane becomes vertical. Taking these limits is a basic sanity check that practicing physicists use every day. Further, it often provides intuition into "what's going on" as you see which pieces of your answer contribute in the different regimes.

3.4 Explain what you *would* do

Sometimes your problem set will be unhappy no matter what you do to cheer it up; while you need to accept these situations, you should not allow your problem set to just wallow in its sorrow.

If you get stuck on a particular part of a problem, do not simply give up. Go as far as you can with that part, then explain why you are stuck (for example, "I don't know how to evaluate this equation for x"). Continue with the rest of the problem and describe what you would do if you could get past the obstacle. I do give partial credit for such descriptions. Further, the process of writing down why you are stuck may clarify the problem and help you solve it.

4 Examples

In this section I present three different solutions to a problem. After each problem comes a discussion of what grade that solution would receive and why. These examples should make very clear what sort of solutions I expect.

4.1 Problem statement

One liter of oil is spilled onto a smooth lake. If the oil spreads out uniformly until it makes an oil slick just one molecule think, estimate the area of the oil slick. Assume the oil molecules have a diameter of 2×10^{-10} cm.

4.2 Solution 1

Before we can figure out the diameter of the slick, we need to know how many oil molecules we have. If $V_{\text{tot}} = 1\text{ L} = 1000\text{ cm}^3$ is the total volume of the oil, and V_{mol} is the volume of one molecule, then the number, N of molecules is

$$N = \frac{V_{\text{tot}}}{V_{\text{mol}}}.$$

If we assume that the molecule is spherical, then its volume is given by

$$V_{\text{mol}} = \frac{4}{3}\pi r^3 = \frac{\pi D^3}{6},$$

where r is the radius and D is the diameter of the molecule. The total area A_{tot} of the slick would then be about the cross-sectional area A_{mol} of one molecule times the number of molecules, or $A_{\text{tot}} = NA_{\text{mol}}$. Since we know the diameter of the molecule, we know that

$$A_{\text{mol}} = \pi r^2 = \pi \frac{D^2}{4}.$$

Putting it all together, our estimate is

$$A_{\text{tot}} = NA_{\text{mol}} = \left(\frac{V_{\text{tot}}}{V_{\text{mol}}}\right) A_{\text{mol}} = V_{\text{tot}} \left(\frac{6}{\pi D^3}\right)\left(\pi\frac{D^2}{4}\right) = \frac{3}{2}\left(\frac{V_{\text{tot}}}{D}\right).$$

Plugging in some numbers gives:

$$A_{\text{tot}} = \frac{3}{2}\left(1000\text{ cm}^3 \times \frac{1\text{ m}^3}{(100\text{ cm})^3}\right)\left(\frac{1}{2.2 \times 10^{-10}\text{ m}}\right) = 6.8 \times 10^6\text{ m}^2$$

4.3 Comments on Solution 1

The above solution is perfect and would score 10/10. The overall logic is clearly expressed in English (Section 2.1); the relationship between each equation is indicated in words, resulting in a high E/M ratio (Section 2.2); all variables, including the standard ones V and A, are defined as they are used (Section 2.4); the problem is solved symbolically all the way through (Section 2.6); the correct number of significant digits is used throughout (Section 2.7); and units are written everywhere and converted correctly (Section 2.9). The above solution, with an incorrect final number, would still get a 9/10. With the logic clear and symbolic answer given, I would know that the student simply punched the wrong numbers into the calculator. This is the sort of solution to strive for.

4.4 Solution 2

$$V = At \quad A = \frac{V}{t} = \frac{0.001}{2.2 \times 10^{-10}} = 4.54545455\text{E}6$$

4.5 Comments on Solution 2

While this solution comes up with a reasonable estimate (which is quite close to the estimate of Solution 1), this solution would only get 3/10. Its cardinal sin is that provides no explanation and thus has an E/M ratio of 0! (Sections 2.1 & 2.2) It further fails to define any variables (Section 2.4); omits units entirely (Section 2.9); and has a ridiculous number of significant digits in the final answer (Section 2.7). Reading this solution later, the student might be hard pressed to explain the thought process that went into it ("Hmmm. What *did* the variable t stand for?"). This is the sort of solution to avoid. I wish to stress that the solution is correct—indeed, an argument could be made that the method of solution is superior to the method of Solution 1. It is the *presentation* which is the problem.

4.6 Solution 3

We know the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is its radius. We solve for r to get

$$r = \sqrt[3]{\frac{3V}{4\pi}} = 6.20 \text{ cm}.$$

We also know that area $A = \pi r^2$, so the answer is 120 cm^2.

4.7 Comments on Solution 3

This solution has some problems. First and foremost, it is getting an area of less than one square meter as opposed to one in the millions of square meters. The reason it is off by so much is that it confuses the volume and area of the oil as a whole with the volume and cross-sectional area of individual molecules. Another problem is that the problem is not solved symbolically all the way through (Section 2.6). Still, there is enough English to clearly express the logic of the solution (Sections 2.1 and 2.2). Moreover, all variables are defined (Section 2.4); the correct number of significant digits is used throughout (Section 2.7); and units are written everywhere (Section 2.9). This solution would receive 5/10. Notice that this is higher than Solution 2, which got a "correct" answer but with a very poor write-up.

5 Final Thoughts

Solving a problem set demands a great deal of effort. The foregoing rules are intended to help you reap the maximum benefit from that effort, in terms of both your mastery of the material and your grade. Should you have any questions about any of the guidelines, or about how a particular problem was graded, you should not hesitate to come ask me.

Good luck on your problem sets!

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