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HW02_CH03

Due: 11:27pm on Wednesday, September 16, 2009

Note: To understand how points are awarded, read your instructor's Grading Policy.

Conceptual Problem about Projectile Motion

Description: Conceptual questions about speed, peak characteristics, flight time, and range for an object undergoing projectile motion.

Learning Goal: To understand projectile motion by considering horizontal constant velocity motion and vertical constant acceleration motion independently.

Projectile motion refers to the motion of unpowered objects (called projectiles) such as balls or stones moving near the surface of the earth under the influence of the earth's gravity alone. In this analysis we assume that air resistance can be neglected.

An object undergoing projectile motion near the surface of the earth obeys the following rules:

1. An object undergoing projectile motion travels horizontally at a constant rate. That is, the x component of its velocity, , is constant.
2. An object undergoing projectile motion moves vertically with a constant downward acceleration whose magnitude, denoted by , is equal to 9.80 near the surface of the earth. Hence, the y component of its velocity, , changes continuously.
3. An object undergoing projectile motion will undergo the horizontal and vertical motions described above from the instant it is launched until the instant it strikes the ground again. Even though the horizontal and vertical motions can be treated independently, they are related by the fact that they occur for exactly the same amount of time, namely the time the projectile is in the air.

The figure shows the trajectory (i.e., the path) of a ball undergoing projectile motion over level ground. The time corresponds to the moment just after the ball is launched from position and . Its launch velocity, also called the initial velocity, is .

Two other points along the trajectory are indicated in the figure.

- One is the moment the ball reaches the peak of its trajectory, at time with velocity . Its position at this moment is denoted by or since it is at its maximum height.
- The other point, at time with velocity , corresponds to the moment just before the ball strikes the ground on the way back down. At this time its position is , also known as (since it is at its maximum horizontal range.

Projectile motion is symmetric about the peak, provided the object lands at the same vertical height from which is was launched, as is the case here. Hence .

Part A

How do the speeds , , and (at times , , and) compare?

ANSWER:

= = > 0
= > = 0
= > > 0
> > > 0
> > = 0

Here equals by symmetry and both exceed . This is because and include vertical speed as well as the constant horizontal speed.

Consider a diagram of the ball at time . Recall that refers to the instant just after the ball has been launched, so it is still at ground level ( ). However, it is already moving with initial velocity , whose magnitude is and direction is counterclockwise from the positive x direction.

Part B

What are the values of the intial velocity vector components and (both in) as well as the acceleration vector components and (both in)? Here the subscript 0 means "at time ."

Hint B.1 Determining components of a vector that is aligned with an axis

If a vector points along a single axis direction, such as in the positive x direction, its x component will be its full magnitude, whereas its y component will be zero since the vector is perpendicular to the y direction. If the vector points in the negative x direction, its x component will be the negative of its full magnitude.

Hint B.2 Calculating the components of the initial velocity

Notice that the vector points up and to the right. Since "up" is the positive y axis direction and "to the right" is the positive x axis direction, and will both be positive.

As shown in the figure, , , and are three sides of a right triangle, one angle of which is . Thus and can be found using the definition of the sine and cosine functions given below. Recall that and and note that

,

What are the values of and ?

Enter your answers numerically in meters per second separated by a comma.

ANSWER:

ANSWER: 15.0, 26.0, 0, -9.80

Also notice that at time , just before the ball lands, its velocity components are (the same as always) and (the same size but opposite sign from by symmetry). The acceleration at time will have components (0, -9.80), exactly the same as at , as required by Rule 2.

The peak of the trajectory occurs at time . This is the point where the ball reaches its maximum height . At the peak the ball switches from moving up to moving down, even as it continues to travel horizontally at a constant rate.

Part C

What are the values of the velocity vector components and (both in) as well as the acceleration vector components and (both in)? Here the subscript 1 means that these are all at time .

ANSWER: 15.0, 0, 0, -9.80

At the peak of its trajectory the ball continues traveling horizontally at a constant rate. However, at this moment it stops moving up and is about to move back down. This constitutes a downward-directed change in velocity, so the ball is accelerating downward even at the peak.

The flight time refers to the total amount of time the ball is in the air, from just after it is launched ( ) until just before it lands ( ). Hence the flight time can be calculated as , or just in this particular situation since . Because the ball lands at the same height from which it was launched, by symmetry it spends half its flight time traveling up to the peak and the other half traveling back down. The flight time is determined by the initial vertical component of the velocity and by the acceleration. The flight time does not depend on whether the object is moving horizontally while it is in the air.

Part D

If a second ball were dropped from rest from height , how long would it take to reach the ground? Ignore air resistance.

Hint D.1 Kicking a ball of cliff; a related problem

Consider two balls, one of which is dropped from rest off the edge of a cliff at the same moment that the other is kicked horizontally off the edge of the cliff. Which ball reaches the level ground at the base of the cliff first? Ignore air resistance.

Hint D.1.1 Comparing position, velocity, and acceleration of the two balls

Both balls start at the same height and have the same initial y velocity ( ) as well as the same acceleration (downward). They differ only in their x velocity (one is zero, the other nonzero). This difference will affect their x motion but not their y motion.

ANSWER:

The ball that falls straight down strikes the ground first.
The ball that was kicked so it moves horizontally as it falls strikes the ground first.
Both balls strike the ground at the same time.

The fact that one ball moves horizontally as it falls does not influence its vertical motion. Hence both balls are at the same height at all moments in time and thus they strike the ground at the same instant.

Now return to the original question, in which you are asked to compare the flight time for a ball that rises from the ground to a peak and then falls back down to the ground with the flight time for a second ball that only needs to fall from the peak height to the ground.

Check all that apply.

ANSWER:

In projectile motion over level ground, it takes an object just as long to rise from the ground to the peak as it takes for it to fall from the peak back to the ground.

The *range* of the ball refers to how far it moves horizontally, from just after it is launched until just before it lands. Range is defined as , or just in this particular situation since .

Range can be calculated as the product of the flight time and the *x* component of the velocity (which is the same at all times, so). The value of can be found from the launch speed and the launch angle using trigonometric functions, as was done in Part B. The flight time is related to the initial *y* component of the velocity, which may also be found from and using trig functions.

The following equations may be useful in solving projectile motion problems, but these equations apply only to a projectile launched over level ground from position ( ) at time with initial speed and launch angle measured from the horizontal. As was the case above, refers to the flight time and refers to the range of the projectile.

flight time:

range:

In general, a high launch angle yields a long flight time but a small horizontal speed and hence little range. A low launch angle gives a larger horizontal speed, but less flight time in which to accumulate range. The launch angle that achieves the maximum range for projectile motion over level ground is 45 degrees.

Part E

Which of the following changes would increase the range of the ball shown in the original figure?

Check all that apply.

ANSWER:

Increase above 30 .
Reduce below 30 .
Reduce from 60 to 45 .
Reduce from 60 to less than 30 .
Increase from 60 up toward 90 .

A solid understanding of the concepts of projectile motion will take you far, including giving you additional insight into the solution of projectile motion problems numerically. Even when the object does not land at the same height from which is was launched, the rules given in the introduction will still be useful.

Recall that air resistance is assumed to be negligible here, so this projectile motion analysis may not be the best choice for describing things like frisbees or feathers, whose motion is strongly influenced by air. The value of the gravitational free-fall acceleration is also assumed to be constant, which may not be appropriate for objects that move vertically through distances of hundreds of kilometers, like rockets or missiles. However, for problems that involve relatively dense projectiles moving close to the surface of the earth, these assumptions are reasonable.

Adding and Subtracting Vectors Conceptual Question

Description: Conceptual question on understanding addition, subtraction, and components of vectors by having them visually compare various combinations of vectors.

Six vectors (A to F) have the magnitudes and directions indicated in the figure.

Part A

Which two vectors, when added, will have the largest x component?

Hint A.1 **Largest x component**

The two vectors with the largest x components will, when combined, give the resultant with the largest x component. Keep in mind that positive x components are larger than negative x components.

ANSWER:

C and E
E and F
A and F
C and D
B and D

Part B

Which two vectors, when added, will have the largest y component?

Hint B.1 **Largest y component**

The two vectors with the largest y components will, when combined, give the resultant with the largest y component. Keep in mind that positive y components are larger than negative y components.

ANSWER:

C and D
A and F
E and F
A and B
E and D

Part C

Which two vectors, when *subtracted* (i.e., when one vector is subtracted from the other), will have the largest magnitude?

Hint C.1 **Subtracting vectors**

To subtract two vectors, add a vector with the same magnitude but opposite direction of one of the vectors to the other vector.

ANSWER:

A and F
A and E
D and B
C and D
E and F

Arrow Hits Apple

Description: A variation of trajectory problem; good test of understanding

An arrow is shot at an angle of above the horizontal. The arrow hits a tree a horizontal distance away, at the same height above the ground as it was shot. Use for the magnitude of the acceleration due to gravity.

Part A

Find , the time that the arrow spends in the air.

Hint A.1 **Find the initial upward component of velocity in terms of D.**

Introduce the (unknown) variables and for the initial components of velocity. Then use kinematics to relate them and solve for . What is the vertical component of the initial velocity?

Hint A.1.1 **Find**

Find the horizontal component of the initial velocity.

Express your answer symbolically in terms of and given symbolic quantities.

ANSWER:

=

Hint A.1.2 **Find**

What is the vertical component of the initial velocity?

Express your answer symbolically in terms of .

ANSWER: =

Express your answer symbolically in terms of and .

ANSWER: =

Hint A.2 Find the time of flight in terms of the initial vertical component of velocity.

From the change in the vertical component of velocity, you should be able to find in terms of and .

Hint A.2.1 Find

When applied to the y-component of velocity, in this problem the formula for with constant acceleration is

What is , the vertical component of velocity when the arrow hits the tree?

Answer symbolically in terms of only.

ANSWER: =

Give your answer in terms of and .

ANSWER: =

Hint A.3 Put the algebra together to find symbolically.

If you have an expression for the initial vertical velocity component in terms in terms of and , and another in terms of and , you should be able to eliminate this initial component to find an expression for

Express your answer symbolically in terms of given variables.

ANSWER: =

Answer numerically in seconds, to two significant figures.

ANSWER: =

Suppose someone drops an apple from a vertical distance of 6.0 meters, directly above the point where the arrow hits the tree.

Part B

How long after the arrow was shot should the apple be dropped, in order for the arrow to pierce the apple as the arrow hits the tree?

Hint B.1 When should the apple be dropped

The apple should be dropped at the time equal to the total time it takes the arrow to reach the tree minus the time it takes the apple to fall 6.0 meters.

Hint B.2 Find the time it takes for the apple to fall 6.0 meters

How long does it take an apple to fall 6.0 meters?

Express your answer numerically in seconds, to two significant figures.

ANSWER: =

Express your answer numerically in seconds, to two significant figures.

ANSWER: =

Projectile Motion Ranking Task

Description: Conceptual question on determining the maximum heights and flight times for projectile motion. (ranking task)

Six baseball throws are shown below. In each case, the ball is thrown with speed at an angle from the horizontal. In all cases, the baseball is thrown from the same height above the ground. Assume for the basis of these rankings that the effects of air resistance are negligible.

Part A

Rank these throws based on the maximum height reached by the ball.

Hint A.1 **Solving two-dimensional motion problems**

A key insight in solving two-dimensional motion problems is the realization that motion in the horizontal direction and motion in the vertical direction are independent. This means that the position, velocity, and acceleration in one direction do not influence the position, velocity, and acceleration in the other direction.

Hint A.2 **Finding vector components**

Given a vector magnitude and angle , the *x* and *y* components of the vector can be determined using the equations and , as shown in the figure.

Rank from largest to smallest. To rank items as equivalent, overlap them.

ANSWER:

View

Part B

Rank these throws based on the amount of time it takes the ball to hit the ground.

Hint B.1 **How to approach the problem**

The time it takes each ball to hit the ground is the sum of the time it takes to reach its maximum height and the time it takes to fall from its maximum height to the ground.

Hint B.2 **Compare times to reach the maximum height**

Consider two balls thrown at different speeds and different angles from the same height. Ball A reaches a maximum height of and ball B reaches a maximum height greater than . Which ball takes longer to reach its maximum height?

ANSWER:

Ball A takes longer to reach its maximum height.
Ball B takes longer to reach its maximum height.
They reach their maximum heights in the same amount of time.
It is impossible to determine without knowing the balls' initial velocities.

Hint B.3 **Compare times to fall from the maximum height**

Consider two balls dropped from different heights. Ball A is dropped from height and ball B is dropped from a height greater than . Which ball takes longer to reach the ground?

ANSWER:

Ball A takes longer.
Ball B takes longer.

They fall in the same amount of time.

Rank from largest to smallest. To rank items as equivalent, overlap them.

ANSWER:

View

Horizontal Cannon on a Cliff

Description: A cannonball is fired horizontally from the edge of a cliff. Three computational questions regarding its projectile motion.

A cannonball is fired horizontally from the top of a cliff. The cannon is at height = 50.0 above ground level, and the ball is fired with initial horizontal speed . Assume acceleration due to gravity to be = 9.80 .

Part A

Assume that the cannon is fired at time and that the cannonball hits the ground at time . What is the *y* position of the cannonball at the time ?

Hint A.1 How to approach the problem

In this problem, you are asked to find the height at a certain time. Nothing is asked or given about the distance coordinate . Therefore, you only need to consider the *y* equations of motion and variables. Write down the known and unknown *y* variables. Then find the appropriate equation(s) and substitute for the values.

Hint A.2 Identify the knowns and unknowns

The information given in the introduction can be used to determine the knowns and unknowns in the problem. For this part, you need to consider only the *y* variables. In terms of the given coordinate system, the initial height can be chosen to be 50.0 . Of course, the acceleration in the *y* direction is exclusively the acceleration due to gravity: .

Which of the following quantities is/are also known?

Check all that apply.

ANSWER:

at time

at time

Thus the initial height is 50.0 , the final height is , and the initial velocity in the *y* direction is .

Hint A.3 Determine which equation to use to find the height at the requested time

Three equations that describe motion in the *y* direction are given below. Which would you use to determine the height of the cannonball at time ?

ANSWER:

When the known vairables are substituted into this equation, you get , where denotes the height at time . In order to find you need to find and substitute for it in this equation .

Hint A.4 Find

What is the value of ?

Hint A.4.1 Identify which equation to use to find

Which of the equations below could you use to find ?

ANSWER:

Substitute the known variables into this equation to find .

Express your answer numerically in seconds.

ANSWER:

=

Now substitute for into the equation for to find at time .

Answer numerically in units of meters.

ANSWER:

=

The same answer can be obtained more easily (perhaps you did it this way) if you notice that . This means that the vertical displacement is given by and therefore is one-quarter of ; then .

Part B

Given that the projectile lands at a distance = 110 from the cliff, as shown in the figure, find the initial speed of the projectile, .

Hint B.1 How to approach the problem

The initial speed can be determined if you know either or and the angle of elevation of the cannon . Then you could use either

.

or

.

In this case, , so there is no component of velocity in the vertical direction, and the second equation is not useful. You need to determine from the given information. In this case, because the cannonball is launched with only an initial horizontal velocity. You are given the horizontal distance traveled by the cannonball and need to find its horizontal velocity (which is constant because the only force acting on the cannonball, gravity, acts exclusively in the vertical direction). You are not asked for or given any information about the *y* variables. Therefore, you need to consider only the *x* variables and equations.

Hint B.2 Knowns and unknowns

Since you are asked to find , you need to determine the knowns and unknowns only for the *x* variables. In the coordinate system shown in the figure in the problem introduction, the known/given *x* variables are

and

110 .

Hint B.3 The equation to use

The equation that describes the motion in the *x* direction is

.

Substitute for the known quantities in this equation and solve for

.

Keep in mind that because the cannonball is launched with an initial horizontal velocity and no initial vertical velocity, and the horizontal component of the cannonball's velocity is constant.

Express the initial speed numerically in meters per second.

ANSWER:

=

Part C

What is the *y* position of the cannonball when it is at distance from the hill? *If you need to*, you can use the trajectory equation for this projectile, which gives in terms of directly:

.

You should already know from the previous part.

Express the position of the cannonball numerically in meters.

ANSWER:

=

Not surprisingly, the answer to this part is the same as that in Part A because a projectile travels equal horizontal distances in equal amounts of time.

Problem 3.9

Description: An airplane is traveling 735 (km)/h in a direction 41.5 degree(s) west of north (see the figure). (a) Find the components of the velocity vector in the northerly and westerly directions. (b) How far north and how far west has the plane traveled…

An airplane is traveling 735 in a direction 41.5 west of north (see the figure).

Part A

Find the components of the velocity vector in the northerly and westerly directions.

Enter your answers numerically separated by a comma.

ANSWER:

, =

Part B

How far north and how far west has the plane traveled after 2.50 ?

Enter your answers numerically separated by a comma.

ANSWER:

, =

Problem 3.16

Description: You are given a vector in the xy plane that has a magnitude of d units and a y component of d_y units. (a) What are the two possibilities for its x component?

You are given a vector in the plane that has a magnitude of 70.0 units and a component of -58.0 units.

Part A

What are the two possibilities for its component?

Enter your answers numerically separated by a comma.

ANSWER:

, = units

Problem 3.26

Description: A hunter aims directly at a target (on the same level) d away. (a) If the bullet leaves the gun at a speed of v, by how much will it miss the target? (b) At what angle should the gun be aimed so as to hit the target?

A hunter aims directly at a target (on the same level) 98.0 away.

Part A

If the bullet leaves the gun at a speed of 181 , by how much will it miss the target?

ANSWER:

=

Part B

At what angle should the gun be aimed so as to hit the target?

ANSWER:

=

Problem 3.35

Description: A rescue plane wants to drop supplies to isolated mountain climbers on a rocky ridge 235 m below. (a) If the plane is traveling horizontally with a speed of v_1 (v_2), how far in advance of the recipients (horizontal distance) must the goods be...

A rescue plane wants to drop supplies to isolated mountain climbers on a rocky ridge 235 below.

Part A

If the plane is traveling horizontally with a speed of 234 (65.0), how far in advance of the recipients (horizontal distance) must the goods be dropped (see the figure)?

ANSWER:

=

Part B

Suppose, instead, that the plane releases the supplies a horizontal distance of 425 in advance of the mountain climbers. What vertical velocity (up or down) should the supplies be given so that they arrive precisely at the climbers' position (see the figure)?

ANSWER:

=

Part C

With what speed do the supplies land in the latter case?

ANSWER:

=

Problem 3.67

Description: A projectile is launched from ground level to the top of a cliff which is 195 m away and 155 m high (see the figure). (a) If the projectile lands on top of the cliff t after it is fired, find the initial velocity of the projectile (magnitude and...

A projectile is launched from ground level to the top of a cliff which is 195 away and 155 high (see the figure).

Part A

If the projectile lands on top of the cliff 5.7 after it is fired, find the initial velocity of the projectile (magnitude and direction). Neglect air resistance.

Express your answer using two significant figures.

ANSWER:

=

Part B

Express your answer using two significant figures.

ANSWER:

=

Score Summary:

Your score on this assignment is 0%.
You received 0 out of a possible total of 10 points.