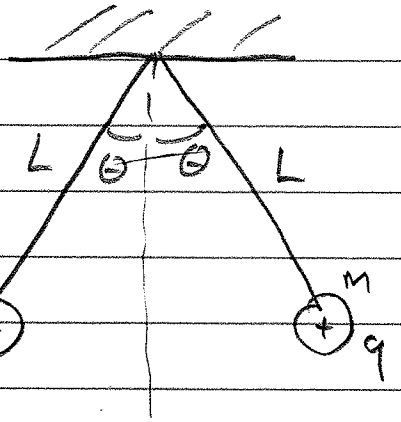
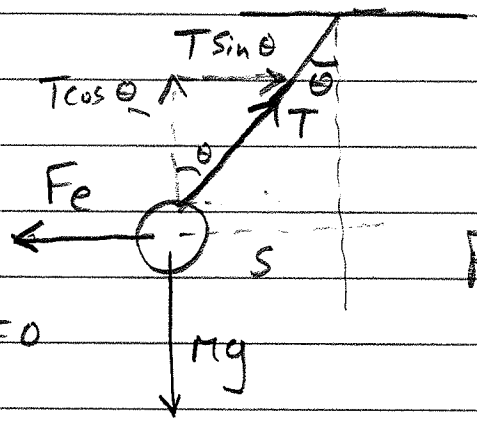
Chap. 23 Example Problems

① Electric Force - Coulomb's Law



System in equilibrium. Find the charge q

Free Body



$s = L \sin\theta$

Vertical

$\Sigma F_y = T\cos\theta - mg = 0$

so $T = \dfrac{mg}{\cos\theta}$

$F_e = k_e \dfrac{q^2}{(2s)^2}$

$= k_e \dfrac{q^2}{4L^2 \sin^2\theta}$

Horizontal

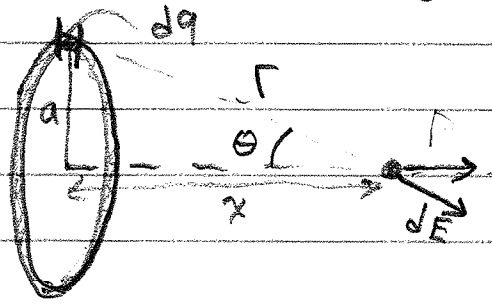
$\Sigma F_x = T\sin\theta - F_e = 0$

$F_e = T\sin\theta = mg \dfrac{\sin\theta}{\cos\theta}$

but $F_e = k_e \dfrac{q^2}{4L^2\sin^2\theta} = mg \dfrac{\sin\theta}{\cos\theta}$

so $q = 2L \sin\theta \sqrt{\dfrac{mg \sin\theta}{k_e \cos\theta}}$

(2) Electric Field along axis of Uniform Ring of Charge.



linear charge density: λ

so total charge:

$$Q = \lambda(2\pi a)$$

For a bit of charge dq,

$$|dE| = k\frac{dq}{r^2}$$ and part along axis

is $$dE_x = k\frac{dq}{r^2}\cos\theta$$

now $r^2 = x^2 + a^2$

and $$\cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+a^2}}$$

so

$$dE_x = k\frac{dq\,x}{(x^2+a^2)^{3/2}}$$

now add up contributions from all dq's → integrate!

$$E = \int dE_x = \int k\frac{dq\,x}{(x^2+a^2)^{3/2}} = \frac{kx}{(x^2+a^2)^{3/2}}\underbrace{\int dq}_{=Q}$$

so

$$E = \frac{k\,2\pi a\,\lambda x}{(x^2+a^2)^{3/2}} \quad \text{or} \quad \frac{kQx}{(x^2+a^2)^{3/2}}$$